Naturalness of Energy Density Functionals

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Skyrme EDF

- Most used EDF in nuclear physics

$$E_t^{even} = C_t^{\rho}\rho_t^2 + C_t^{\tau}\rho_t\tau_t + C_t^{\Delta\rho}\rho_t\Delta\rho_t + C_t^{\nabla J}\rho_t\nabla J_t + C_t^{J}J_t^2$$

$$C_t^{\rho} = C_{t0}^{\rho} + \rho_0^{\gamma}C_{tD}^{\rho} \quad , \quad t = 0,1$$

- Parametrized by 12 coupling constants and one power
- Historically Skyrme force was defined in tx-parametrization
- Density dependence included in ρ_t^2 term
- Contains derivatives of density matrices up to second order

Effective Lagrangian

- Each term in effective low-energy Lagrangian is written as

$$g\left[\frac{\psi\psi^{\dagger}}{\Lambda f_{\pi}^{2}}\right]^{l}\left[\frac{\nabla}{\Lambda}\right]^{n}\Lambda^{2}f_{\pi}^{2}$$

- f_{π}^{2} is a pion-decay constant, g is a dimensionless constant close to unity and Λ characterizes energy scale beyond effective Lagrangian
- Standard Skyrme coupling constants are scaled by a factor of

$$S = f_{\pi}^{2(l-1)}\Lambda^{n+l-2}$$

- Isovector coupling constants in natural units are scaled by a factor of 4
- Continuation of Furnstahl & Hackworth, PRC56, 2875
- Natural units scaling applied to 48 Skyrme functionals

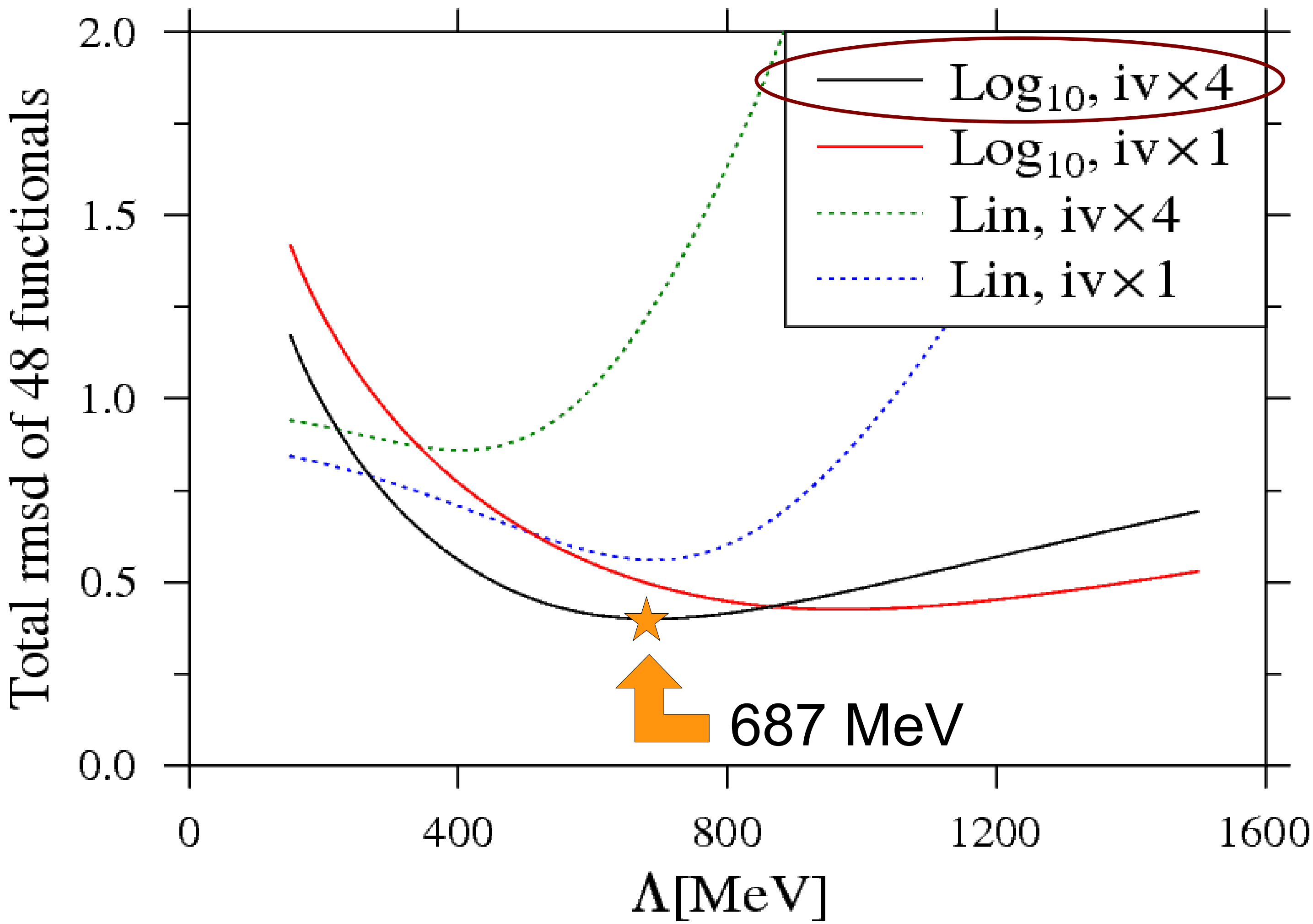
List of functionals

Index	Functionals	Categories
1–2	SkT3, SkT6	a d i j
3	SkM	a c e f g i
4	SkM*	g j
5–6	SGI, SGII	d e j
7	HFB9	a b f h i
8–9	SI, SII	a c d e f i
10	SkA	a c d e g i j
11	HFB16	a b c f h i
12	SkT	a b d e g h i
13–16	SLy4–7	a c d e f i
17–18	SkI1–2	a b c d f g i
19–20	SkI3–4	a b c d f g i k
21	SkI5	a b c d f g i
22–27	MSk1–6	a b f h i
28–29	SIII, SIV	a c i
30–31	SV, SVI	a c i j
32–33	SLy230a,b	a c d e f i
34–39	E, E_σ, Z, Z_σ, R_σ, G_σ	a c d g i
40	SkP	a b c e f h i
41–42	SkO,SkO'	a b c d f g i k
43	SV-min	a b c d g h k
44	$SkO_{T''}$	i j k
45	SkMP	a j
46–47	SkX, SkX_c	a b c d e f
48	RATP	a d e f i

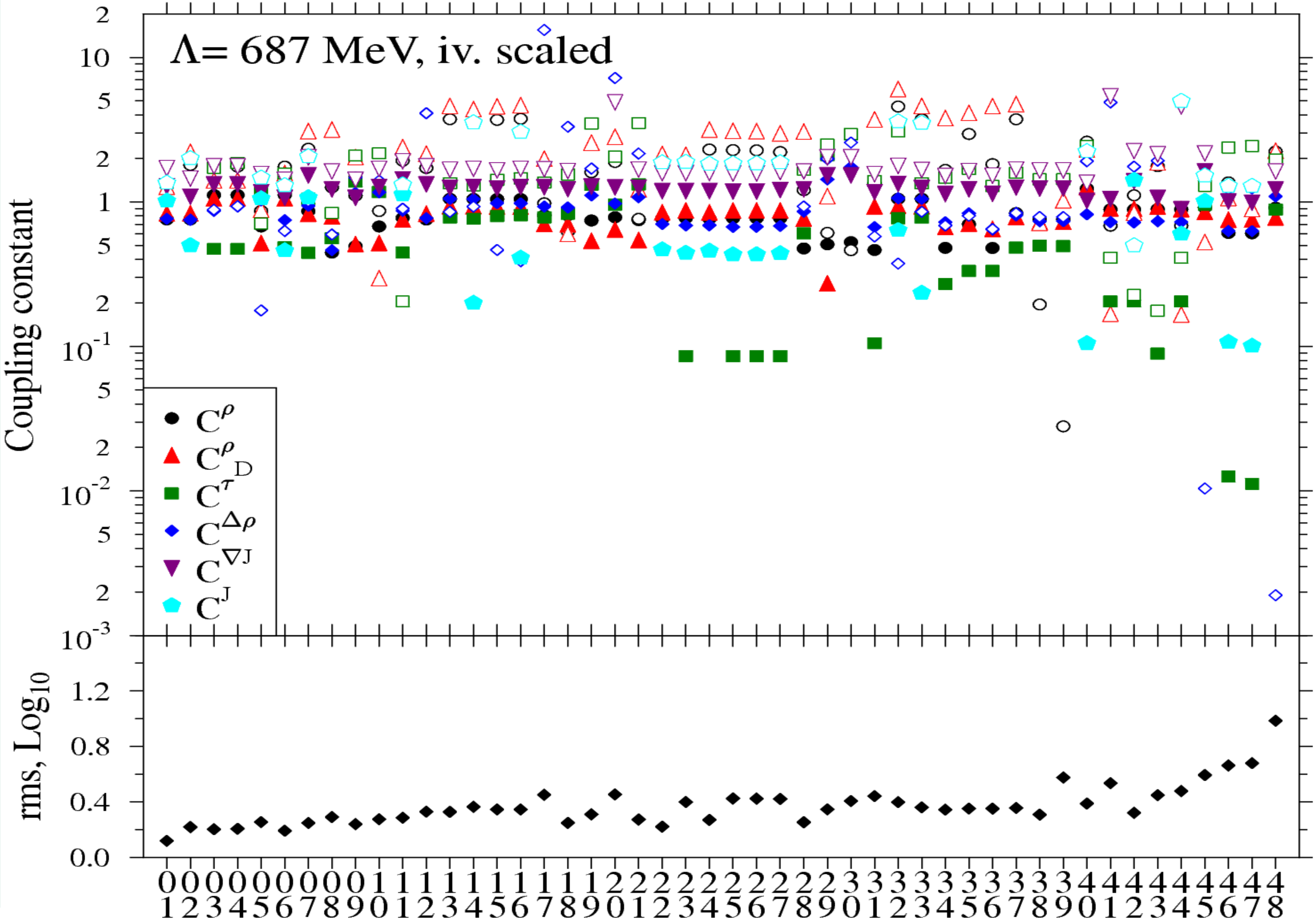
a) masses of double-magic nuclei (includes ^{90}Zr, ^{116}Sn, ^{124}Sn and ^{140}Ce) used in the fit
b) masses of non-double-magic nuclei used in the fit
c) charge radii used in the fit
d) single particle energies used in the fit
e) symmetric infinite nuclear matter constrains considered in the fit
f) asymmetric infinite nuclear matter constrains considered in the fit
g) surface properties (neutron skin, fission barriers etc.) considered in the fit
h) pairing was present in the fit
i) some parameters were fixed in the fit
j) parameters extrapolated or fine-tuned from existing force or functional
k) functional is an extended functional

Choice of optimal Λ

- One way to obtain optimal Λ is to minimize the deviation of coupling constants from unity ⇒ rmsd fit



Coupling constants in natural units



Deviation from unity

- Deviation from unity linked in many cases to uncomplete or specific optimization procedure
- Anomalously small $C^{\Delta\rho}_1$ in RATP explained by a focus on volume part of the functional for astrophysical purposes
- Small $C^{\Delta\rho}_1$ in SkMP comes from mixing SkM* and SkP functionals with only minor adjustment to volume part
- Large $C^{\Delta\rho}_1$ in SkI1 compared to other SkI functionals due to the lack of isotopic shift data in optimization
- Large C^{τ}_0 in SV due to artificially imposed zero density dependence
- Small C^{τ}_0 in SkX results from strong emphasis on single particle data which favors effective mass close to one

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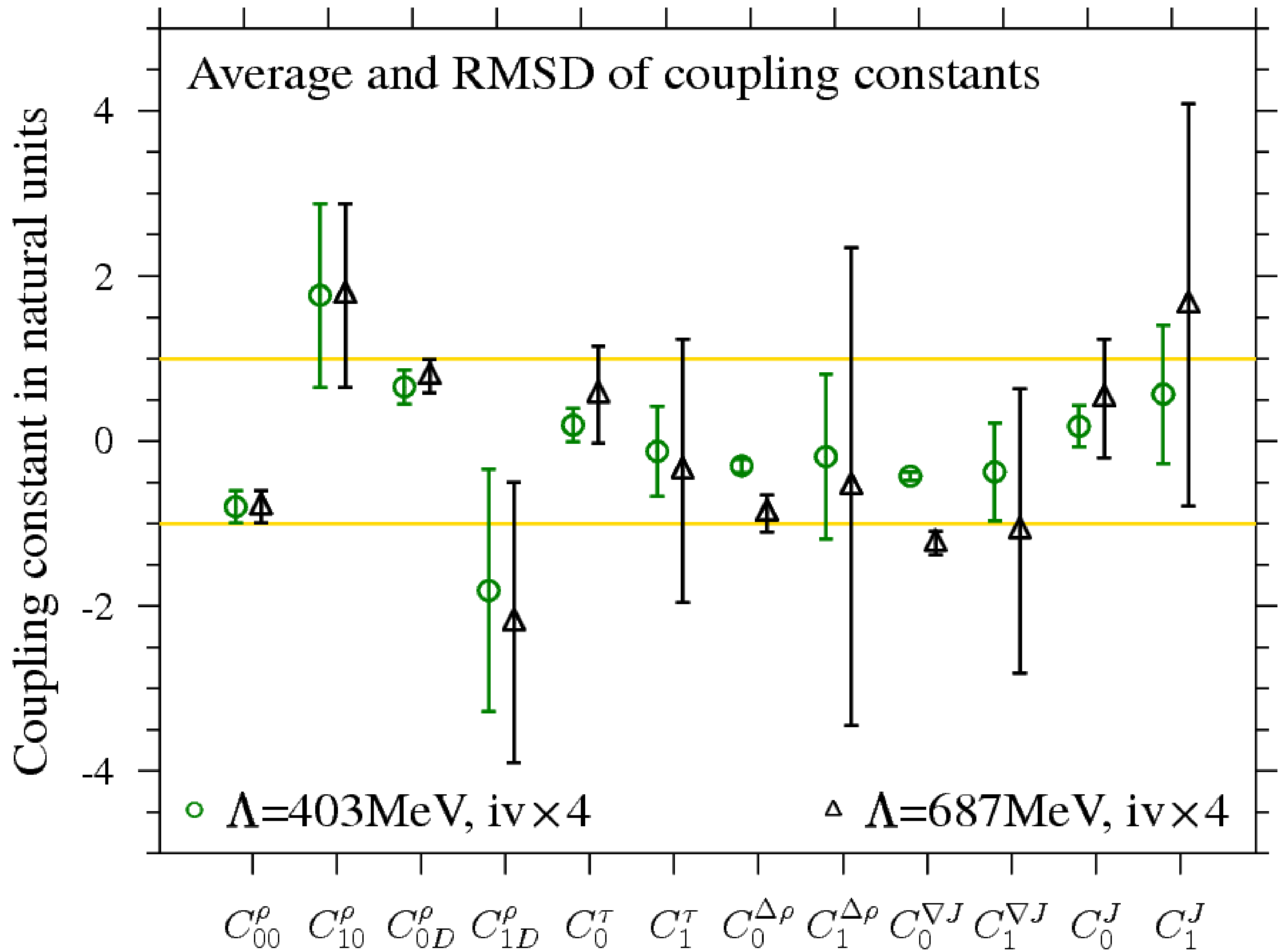
Range of coupling constants in natural units

- Naturalness can be used as a guide in future fitting attempts
- Average and RMSD from average for cc's of 48 functionals:



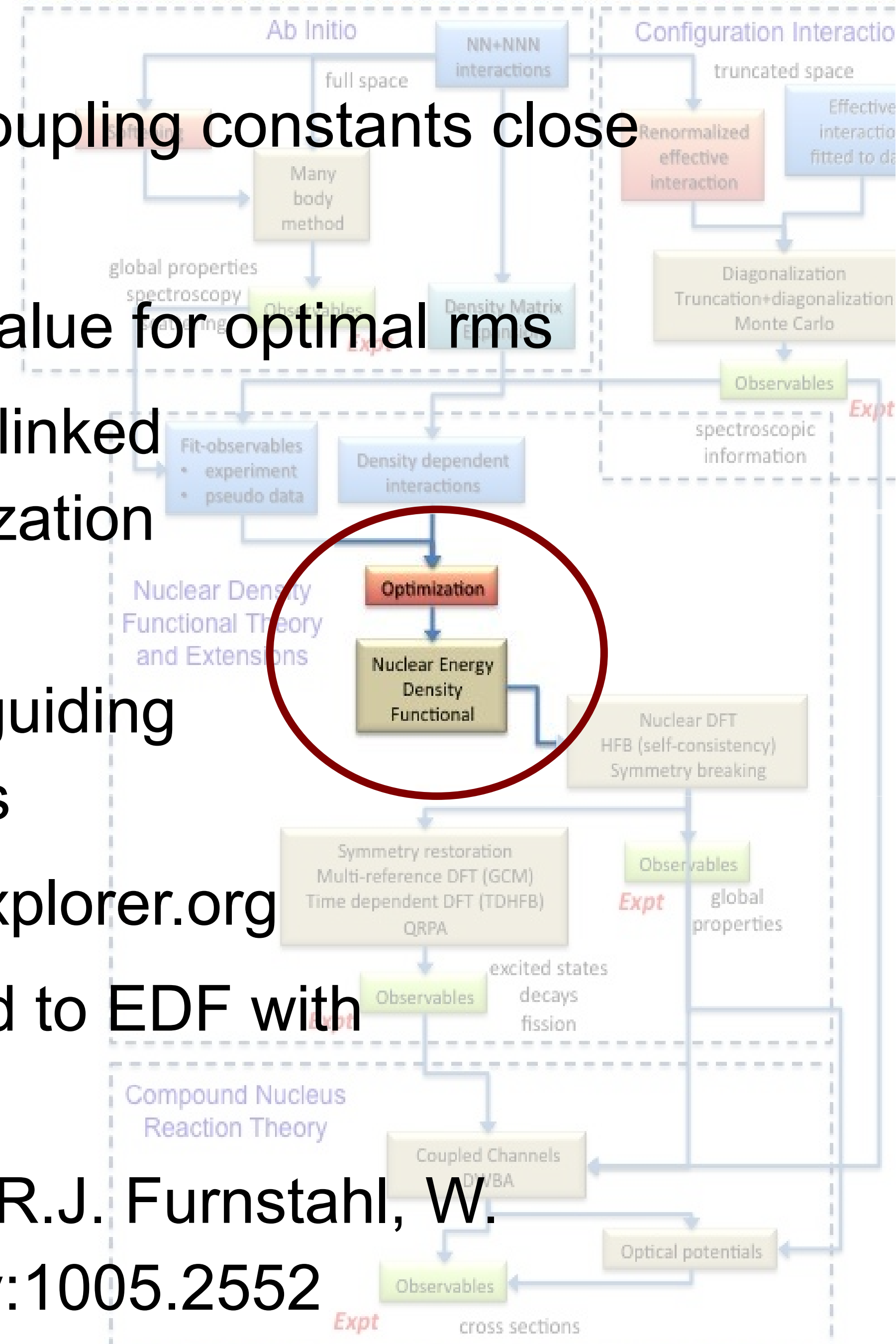
Coupling constants and natural units

- In tx-parametrization similar functionals can have very different looking parameters
- Natural units give better picture of the functional parameters

tx-parametrization			Coupl. cons.			Nat. units	
	SIII	HFB16		SIII	HFB16	SIII	HFB16
t_0	-1128.75	-1837.23	C_{00}^{ρ}	-423.2813	-688.9613	-0.4767	-0.7759
t_1	395	383.521	C_{10}^{ρ}	268.0781	428.3502	1.2076	1.9295
t_2	-95	-3.41736	C_{0D}^{ρ}	875.0000	720.1875	0.7623	0.7509
t_3	14000	11523	C_{1D}^{ρ}	-875.0000	-571.2513	-3.0493	-2.3825
x_0	0.45	0.4326	C_0^{τ}	44.3750	32.6943	0.6059	0.4464
x_1	0	-0.824106	C_1^{τ}	-30.6250	-3.7499	-1.6726	-0.2048
x_2	0	44.652	$C_0^{\Delta\rho}$	-62.9688	-63.7366	-0.8597	-0.8702
x_3	0	0.689797	$C_1^{\Delta\rho}$	17.0313	-16.4752	0.9301	-0.8998
W_0	120	141.1	$C_0^{\nabla J}$	-90.0000	-105.8250	-1.2288	-1.4449
			$C_1^{\nabla J}$	-30.0000	-35.2750	-1.6384	-1.9265
			C_0^{J}	0.0000	82.7654	0.0000	1.1300
			C_1^{J}	0.0000	24.1836	0.0000	1.3208

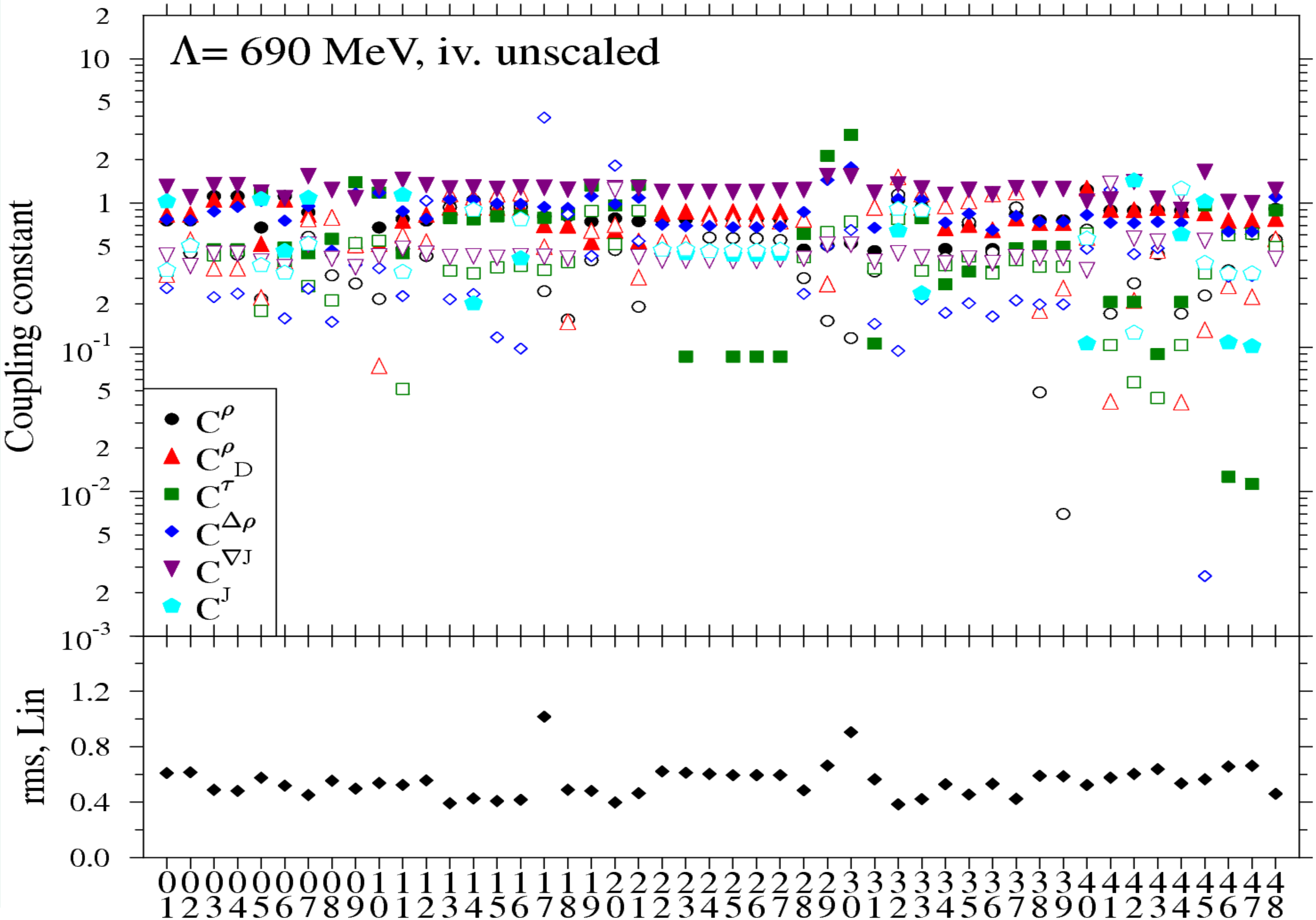
Conclusions

- Scaling to natural units brings coupling constants close to unity
- Parameter Λ around expected value for optimal rms
- Large deviations from unity can linked to uncomplete or specific optimization procedure in many cases
- Natural units can be used as a guiding principle in future fitting attempts
- Natural units included in massexplorer.org
- Natural units can be also applied to EDF with higher order terms
- For details see: M. Kortelainen, R.J. Furnstahl, W. Nazarewicz, M.V. Stoitsov, arXiv:1005.2552



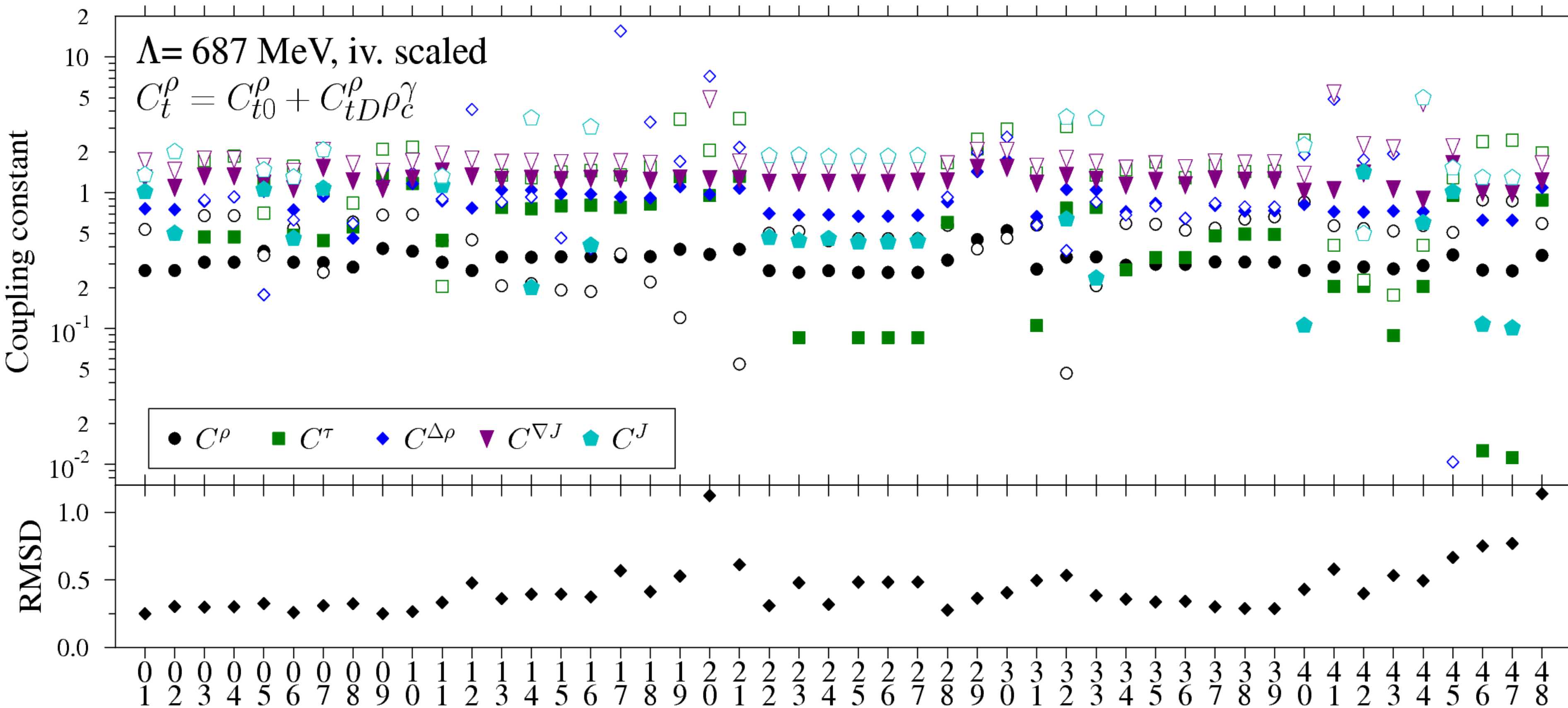
Additional slides

Coupling constants in natural units



Coupling constants in natural units

- Combining C^{ρ} coupling constants into a one density dependent cc. does not improve results (ρ_c = 0.16fm^{-3})



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