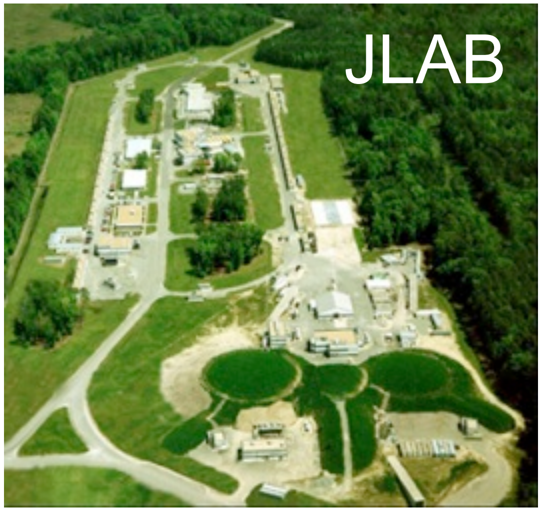
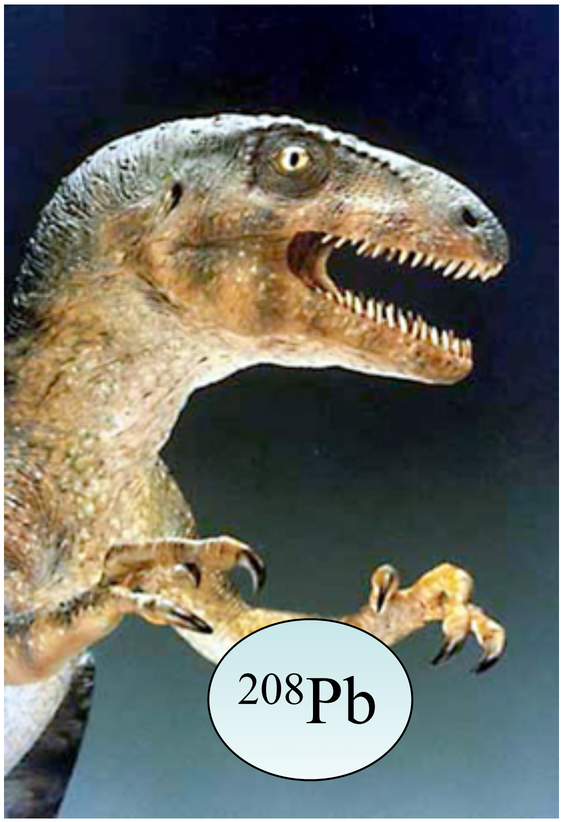
Astrophysical Equations of State

- Pb Radius Experiment (PREX) first result for neutron radius of ^{208}Pb.
- New astrophysical equations of state:
  - Virial expansion with nucleons, alphas + thousands of heavy nuclei at low densities.
  - Extensive relativistic mean field calculations at high densities.



C. J. Horowitz + ***G. Shen***, Indiana University, UNEDF, June 2011

Pb Radius Experiment (PREX)



Provides a precise laboratory probe of neutron rich matter.

PREX at Jefferson Laboratory uses parity violating electron scattering to accurately measure the neutron radius of ^{208}Pb.

This has many implications for nuclear structure, astrophysics, atomic parity violation, and low energy tests of the Standard Model.

Spokespersons: K. Kumar, P. Souder, R. Michaels, G. Urciuoli

Parity Violation Isolates Neutrons

- In Standard Model Z^0 boson couples to the weak charge.
- Proton weak charge is small:

$$Q_W^p = 1 - 4\sin^2\Theta_W \approx 0.05$$

- Neutron weak charge is big:

$$Q_W^n = -1$$

- Weak interactions, at low Q^2, probe neutrons.
- Parity violating asymmetry A_{pv} is cross section difference for positive and negative helicity electrons

$$A_{pv} = \frac{d\sigma/d\Omega_+ - d\sigma/d\Omega_-}{d\sigma/d\Omega_+ + d\sigma/d\Omega_-}$$

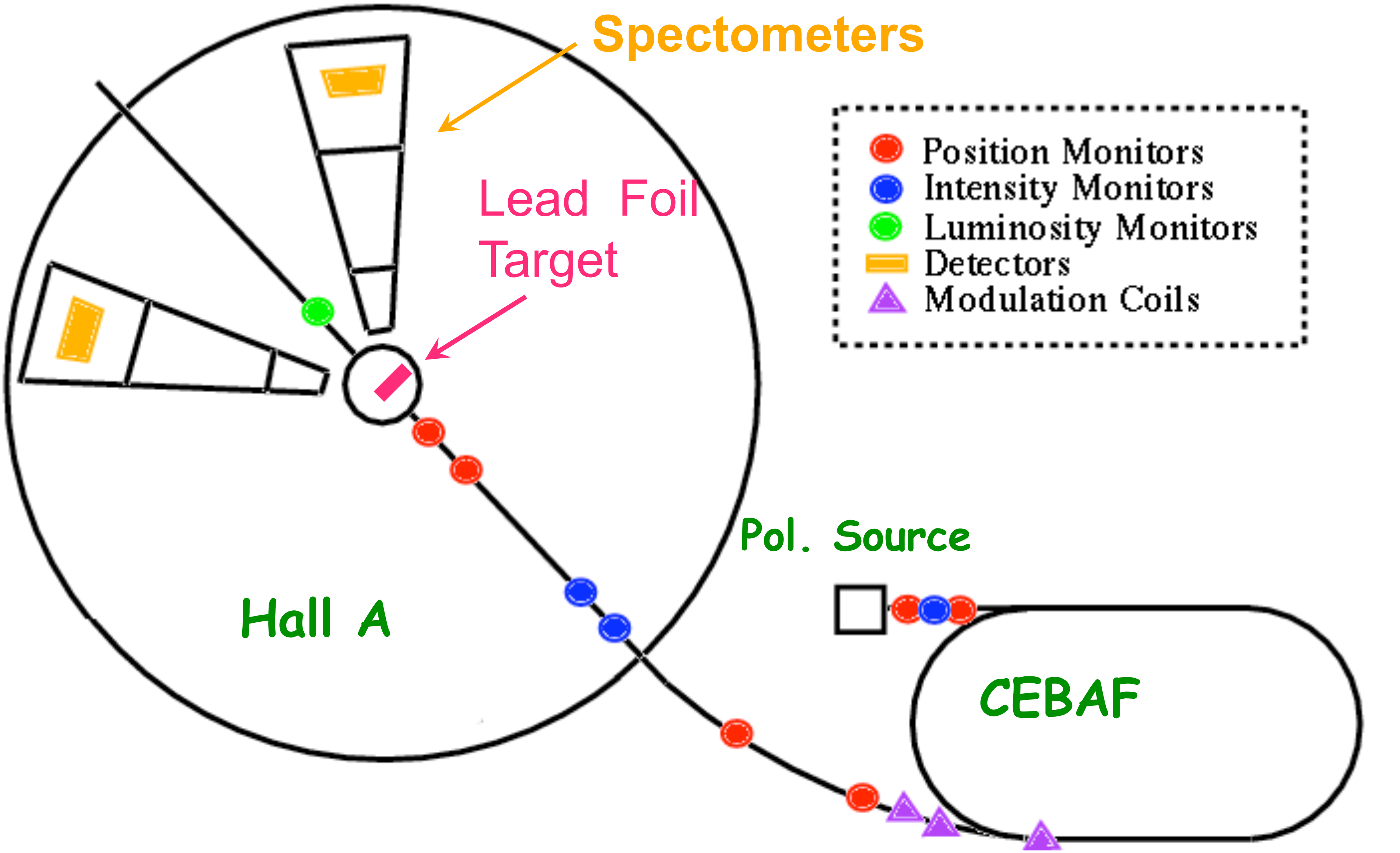
- A_{pv} from interference of photon and Z^0 exchange. In Born approximation

$$A_{pv} = \frac{G_F Q^2}{2\pi\alpha\sqrt{2}} \frac{F_W(Q^2)}{F_{\rm ch}(Q^2)}$$

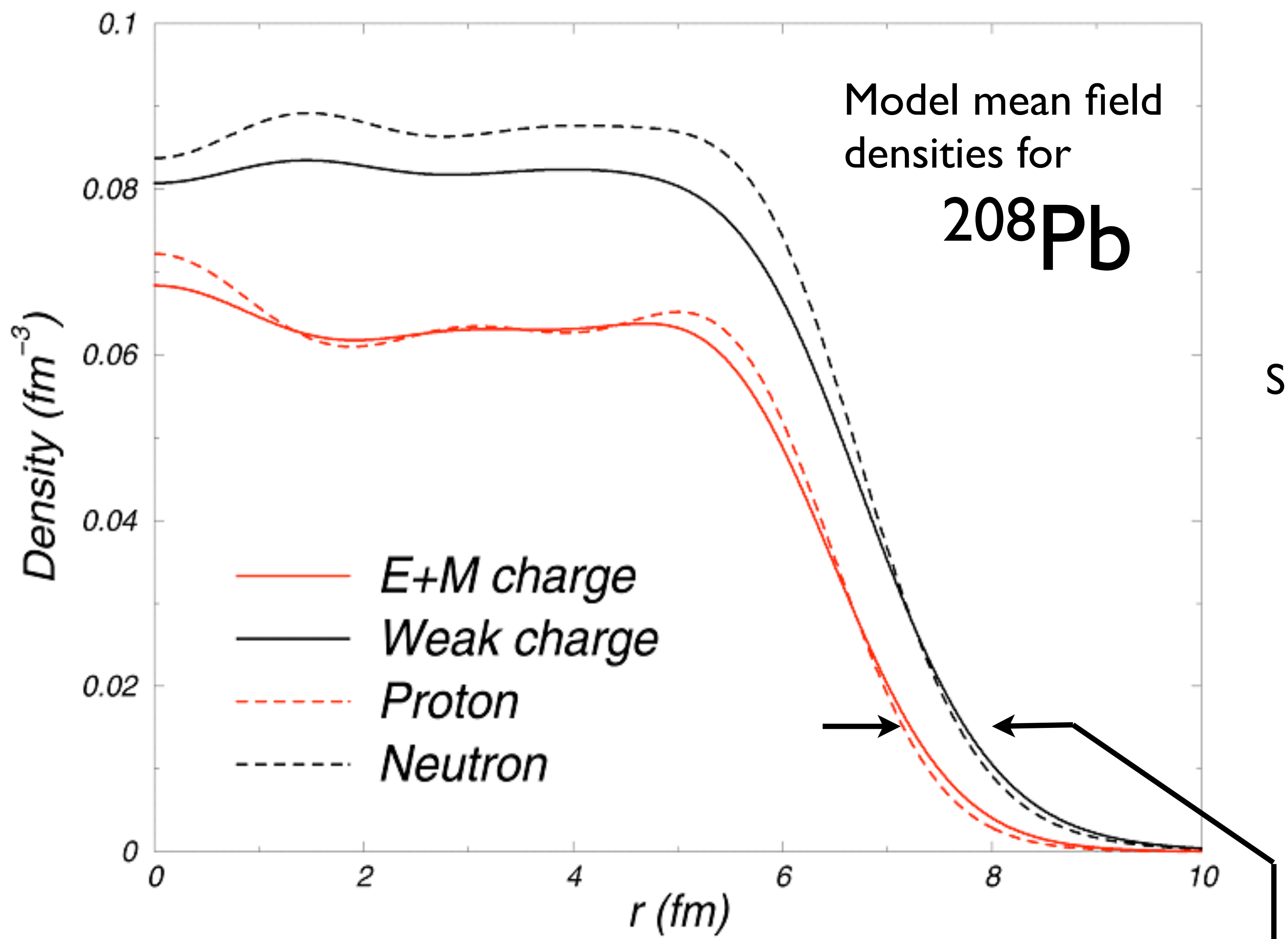
$$F_W(Q^2) = \int d^3r \frac{\sin(Qr)}{Qr} \rho_W(r)$$

- Coulomb distortions important but accurately calculated.
- PREX measure A_{pv} for 1.05 GeV electrons scattering from ^{208}Pb at 5 degrees. Goal measure A_{pv} to 3%, gives neutron radius R_n to 1% (+/- 0.05 fm).
  - Donnelly, Dubach, Sick first suggested PV to measure neutrons.

PREX in Hall A at JLab



R. Michaels



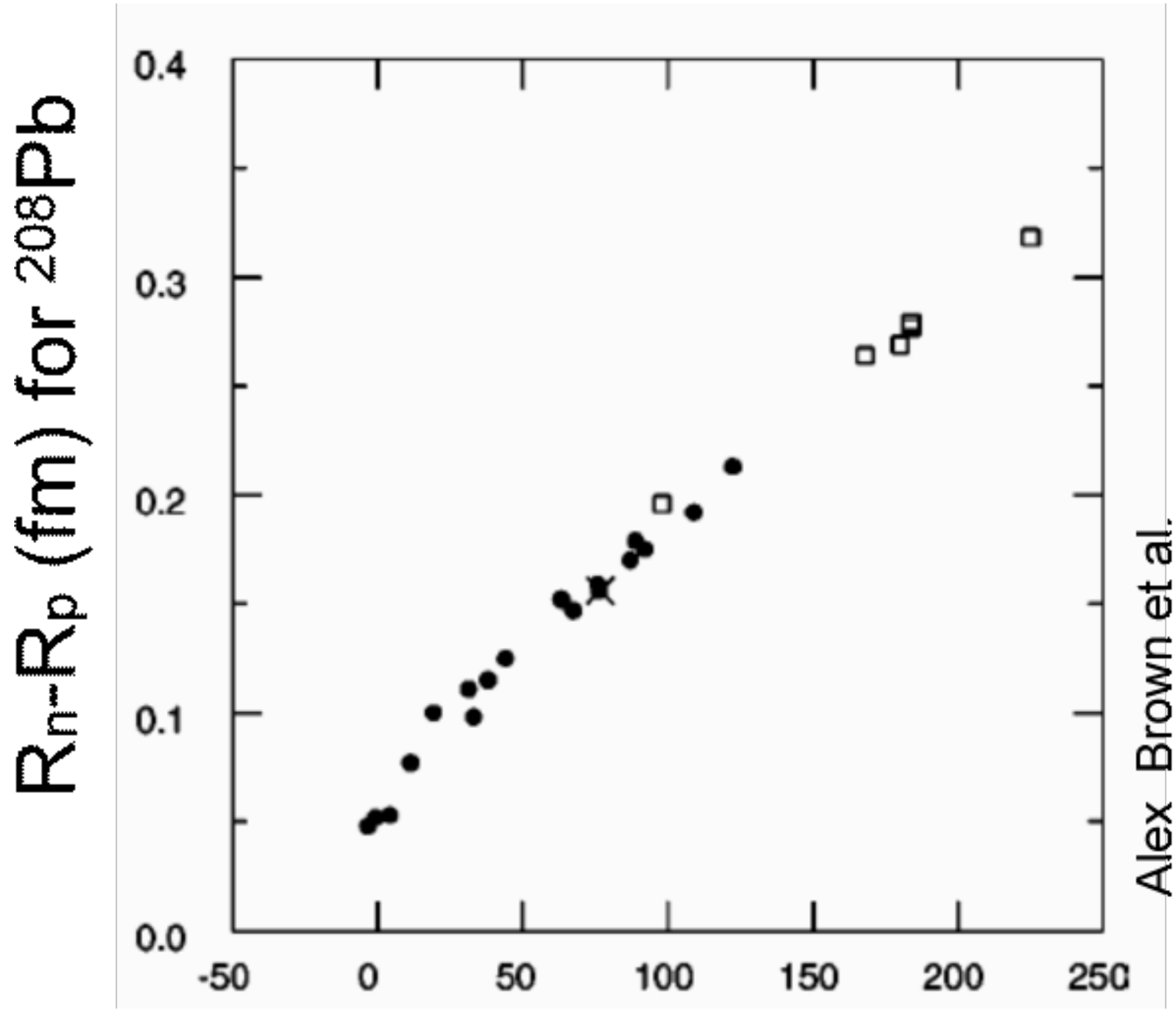
PREX

Spokespersons:
K. Kumar
P. Souder
R. Michaels
G. Urchiuli

- PREX measures how much neutrons stick out past protons (neutron skin).
- First result announced April 30, 2011. Measured parity violating asymmetry: A_{pv} = +0.6571 ± 0.0604 ± 0.0130 ppm implies: R_n-R_p=$0.34^{+0.15}_{-0.17}$ fm
- Plan to run again to obtain more statistics and reach 1% error +/-0.05 fm for R_n, also second measurement in ^{48}Ca very attractive.

^{208}Pb radius and equation of state

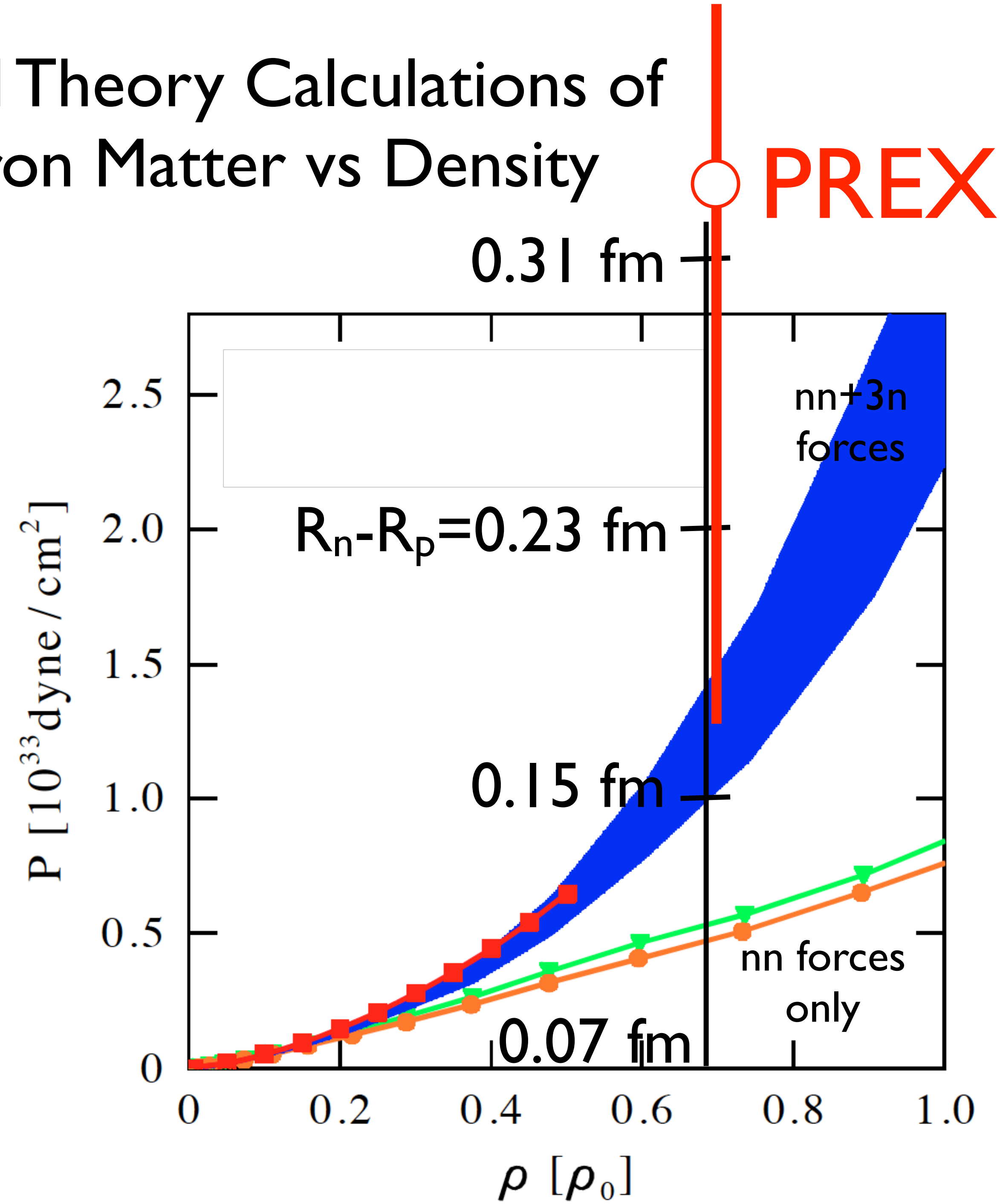
- Pressure of neutron matter forces neutrons out against surface tension. A large pressure gives a large neutron radius.
- Measuring R_n in ^{208}Pb constrains the pressure of neutron matter at $\sim 2/3 \rho_0 = 0.1\ \mathrm{fm}^{-3}$.



Neutron matter P (MeV/fm^3) at a density of 0.1 fm^{-3}.

Chiral Effective Field Theory Calculations of Pressure of Neutron Matter vs Density

- A. Brown found strong correlation between pressure of neutron matter at a density of $0.66\rho_0$ and R_n-R_p in ^{208}Pb (see 2nd vertical scale)
- Chiral EFT calc. by Hebeler et al. with only two n forces are green and brown while blue band shows results including 3 neutron forces. PRL **105**, 161102 (2010)
- PREX agrees with results including 3n forces. Hebeler et al. predict R_n-R_p=0.14 to 0.2 fm.



New Astrophysical Equations of State

- For simulations of supernovae (SN), neutron star mergers, black hole formation...
- Need pressure P as function of density n, temperature T, and proton fraction Y_P over large range (calculated at 180,000 points) $0 < Y_P < 0.56$, $0 < T < 80$ MeV, $10^{-8} < n < 1.6$ fm^{-3}.
- Almost all realistic SN simulations use 1 of 2 EOSs
  1. Lattimer Swesty (LS) based on crude liquid drop model.
  2. H. Shen et al (HShen) based on relativistic mean field model in Thomas Fermi and variational approximations.
- Recently Hempel + Schaffer-Bielich have new EOS based on nuclear statistical model.
- Our EOSs uses extensive relativistic mean field calculations at high densities and virial + statistical model at low densities. EOS tables at http://cecelia.physics.indiana.edu/gang_shen_eos/

Virial + Statistical Model

- At low densities consider nucleons + alphas + thousands of heavy nuclei (A>11).
- Expand pressure in powers of fugacities z_i.

$$\frac{P}{T} = \frac{2}{\lambda_n^3}[z_n + z_p + (z_p^2 + z_n^2)b_n + 2z_p z_n b_{pn}] + \frac{1}{\lambda_\alpha^3}[z_\alpha + z_\alpha^2 b_\alpha + 2z_\alpha(z_n + z_p)b_{\alpha n}] + \sum_i \frac{1}{\lambda_i^3} z_i \Omega_i, \quad (1)$$

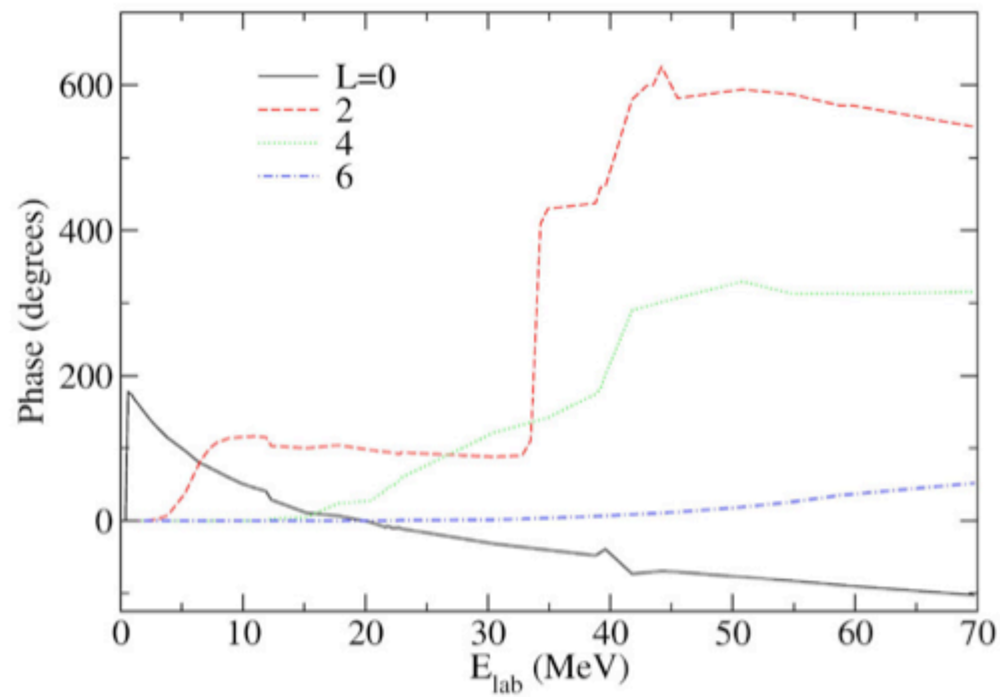
$$z_i = \exp(\mu_i + E_i)/T = z_p^Z z_n^N e^{E_i/T}$$

Ω_i=partition function for heavy nuclei, E_i is binding E.

2nd Virial Coefficients

$$b_2 = 2^{1/2} \sum_B e^{E_B/T} + \frac{2^{1/2}}{\pi} \int_0^\infty dk\, e^{-E_k/2T} \sum_l (2l+1) d\delta_l(k)/dk \pm 2^{-5/2}$$

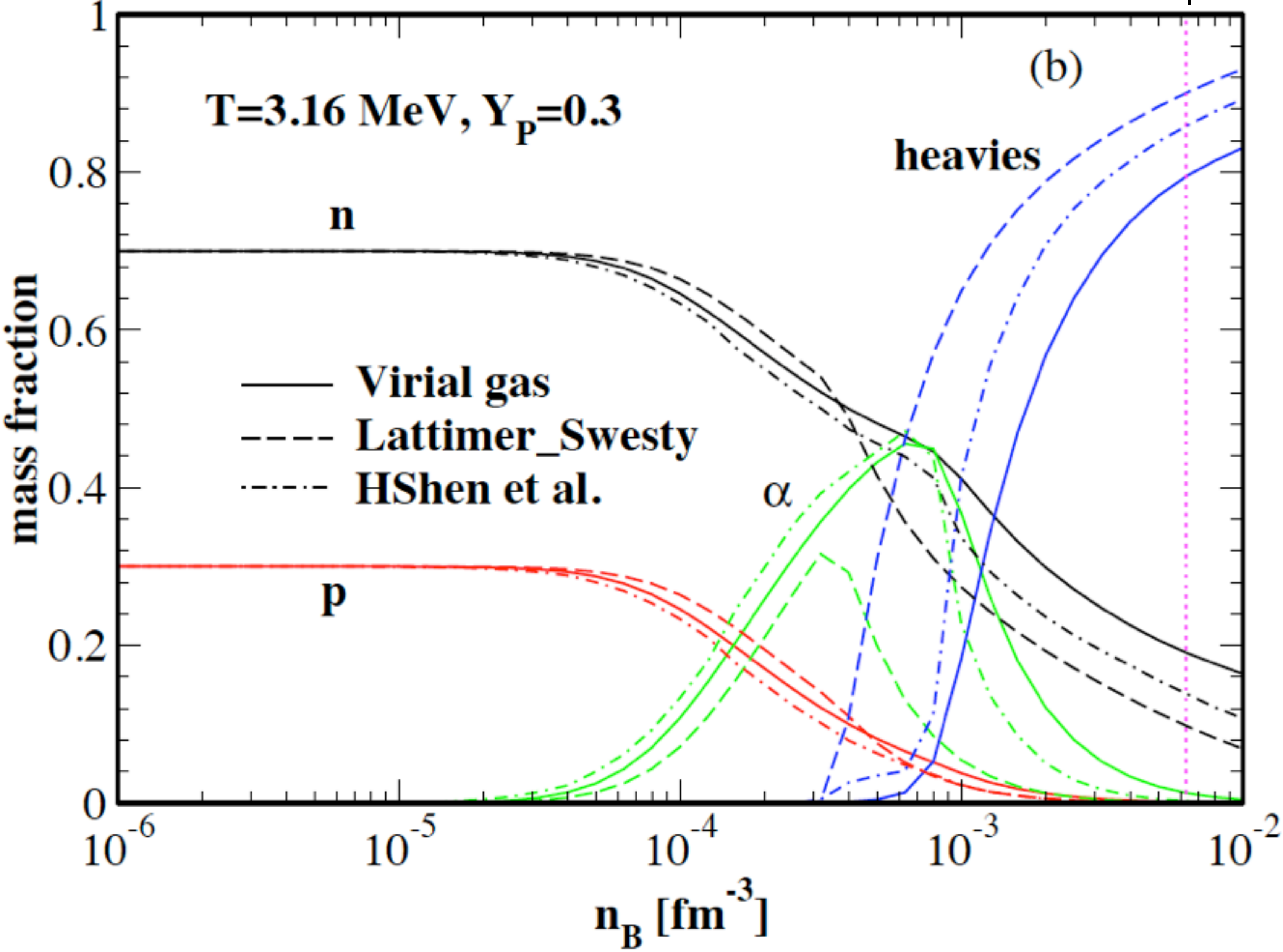
- Virials from NN, Nα, and αα elastic scattering phase shifts.
- Note scattering resonance near zero energy gives same result as zero energy bound state.
- For heavy nuclei we use FRDM mass model.
- Virial corrections reproduce near unitary neutron rich gas at higher entropies.



α−α Elastic Phase Shifts

Composition: nucleons, alphas, heavy nuclei

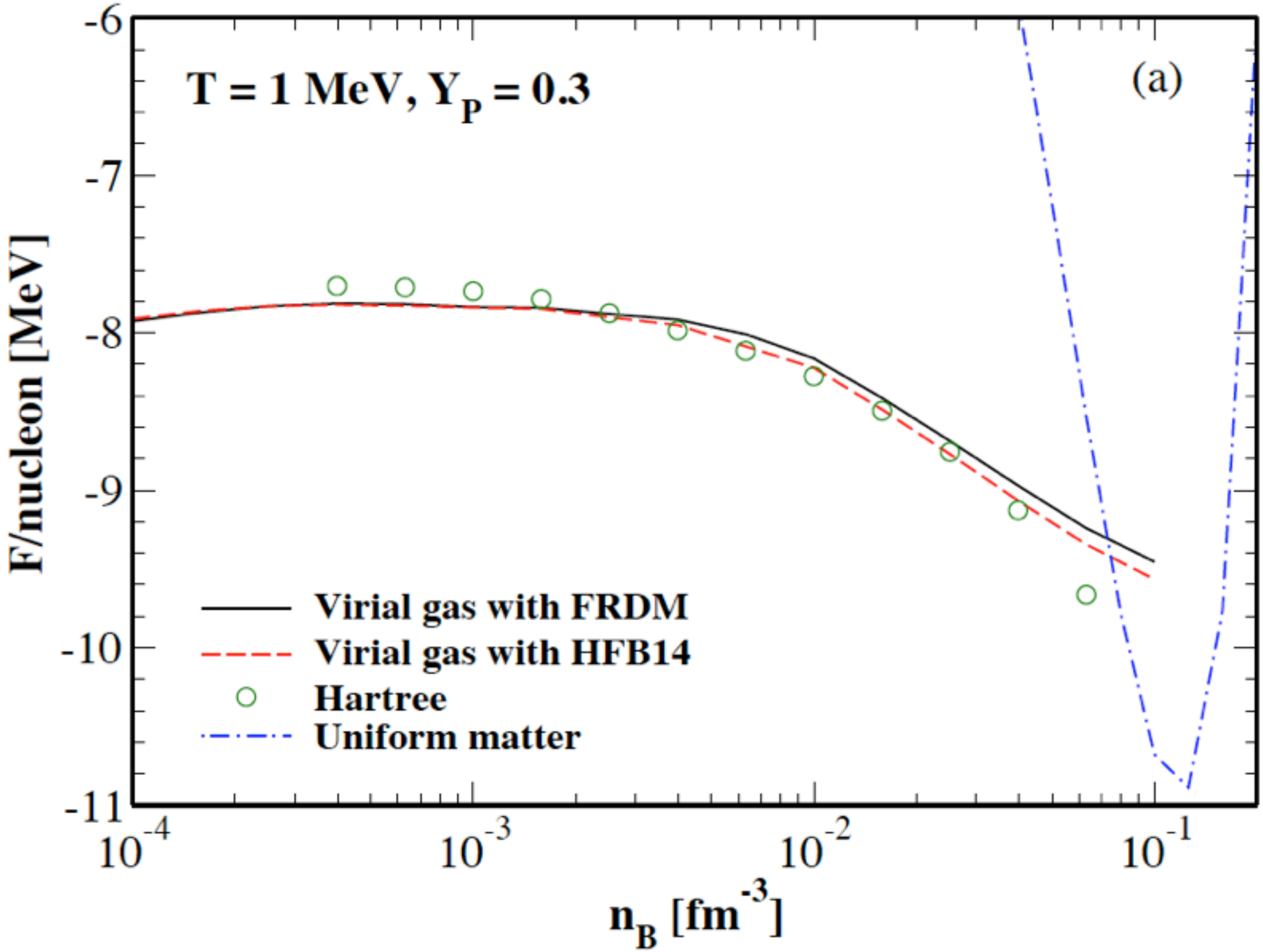
<-- Virial+ NSE | RMF -->



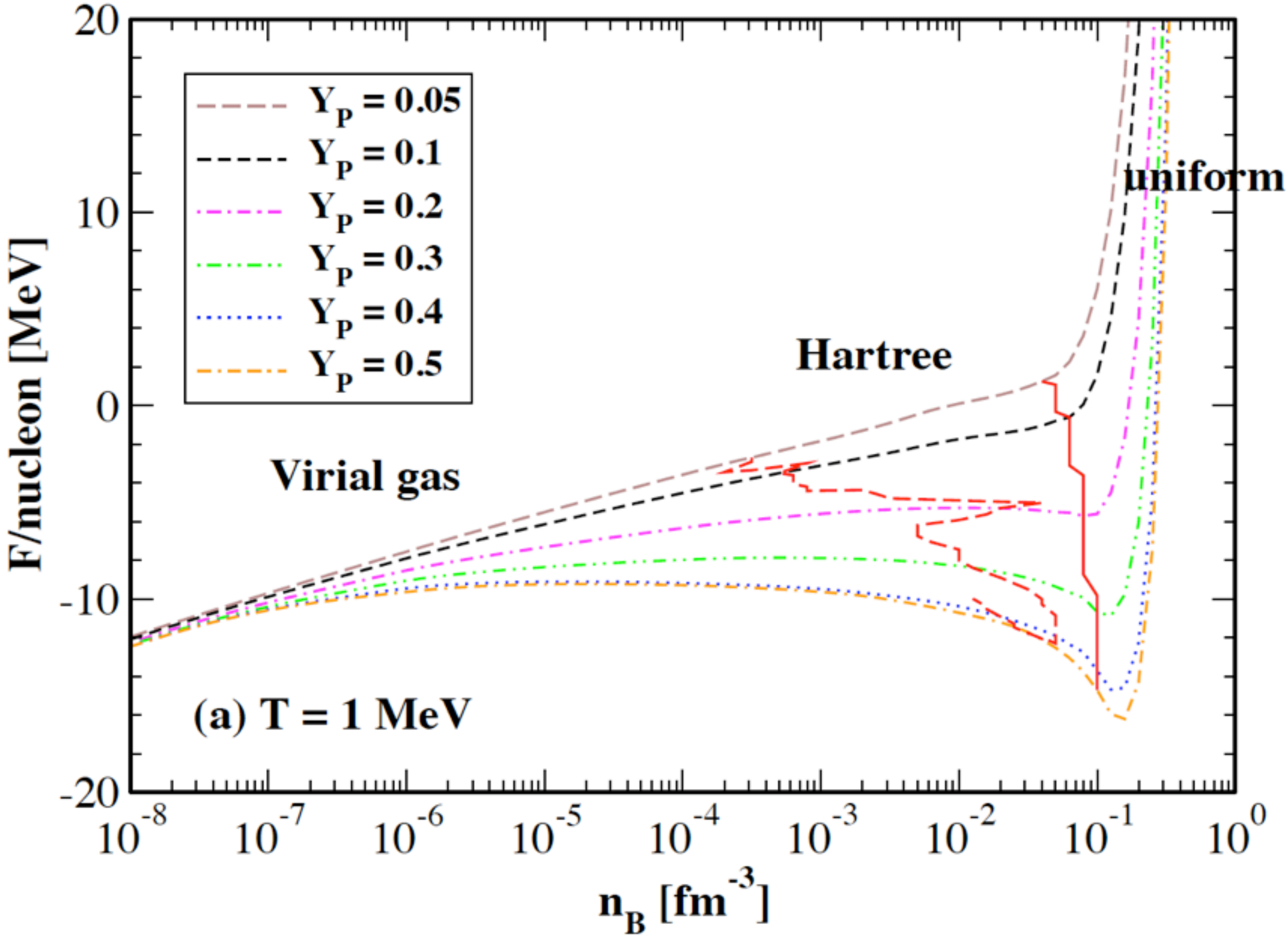
Relativistic Mean Field Calculations

- Spherical Wigner Seitz boundary conditions: given n_B, Y_p minimize free E wrt cell radius R_s.
- Finite temperature T calc. with very many levels.
- Repeat for large numbers of n_B, T, Y_p.
- *About 200,000 CPU hours per EOS*: different cores work on different n_B, T, Y_p.
- Now have three EOSs available, see G. Shen et al., arXiv:1103.5174
  - Stiff: based on NL3 interaction, maximum NS mass is 2.7 M_{sun}. --> NL3
  - Soft: based on FSUgold, maximum mass is 1.7 M_{sun}. --> FSU1.7
  - Medium: modify FSUgold at high densities so that maximum mass is 2.1 M_{sun}. --> FSU2.1

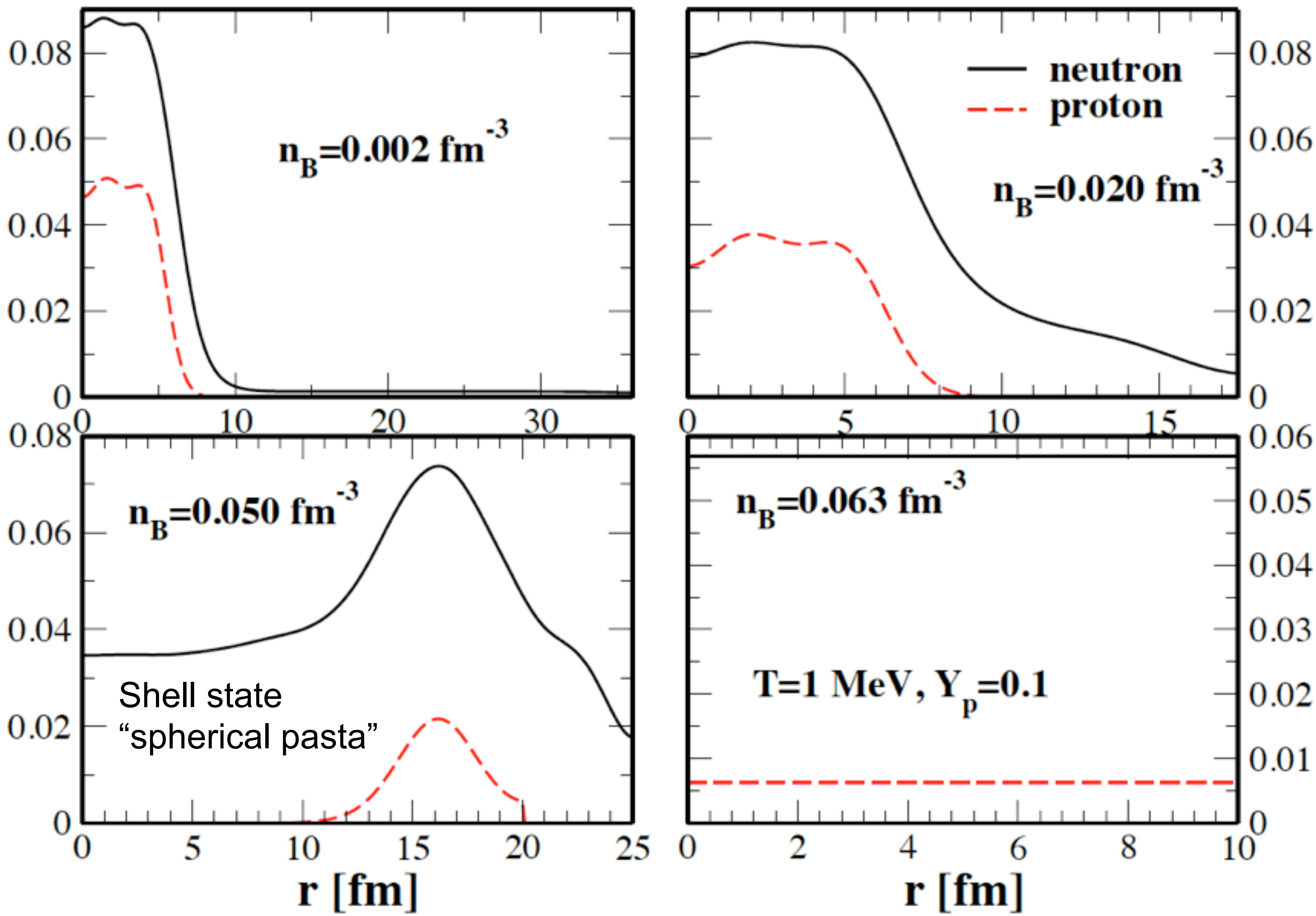
Matching of Virial, Hartree (nonuniform) and Uniform Mean Field Free Energies



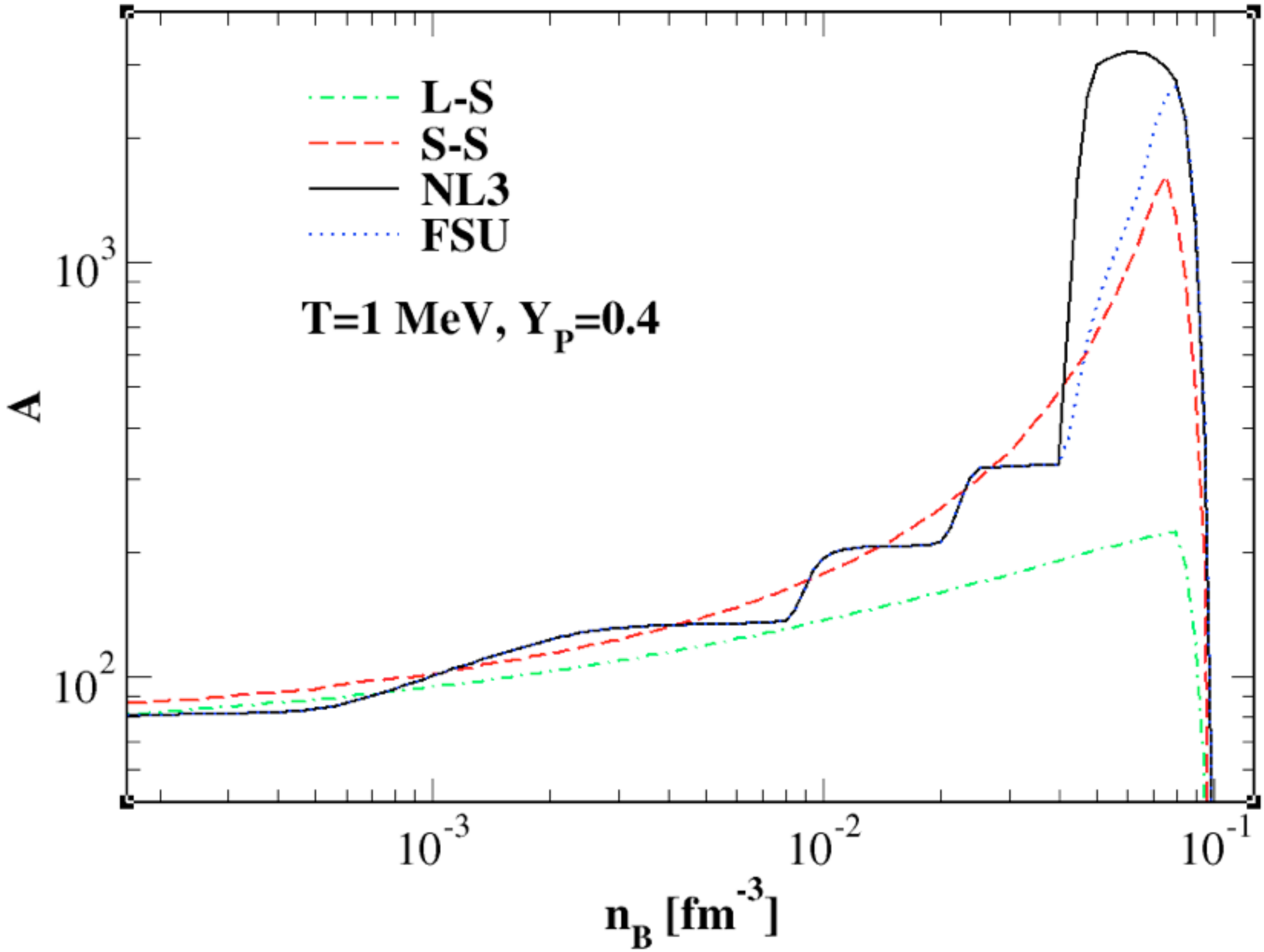
Matching of Virial and Mean Field Free Energies



NL3 Mean Field Density Profiles



Average A of heavy nuclei



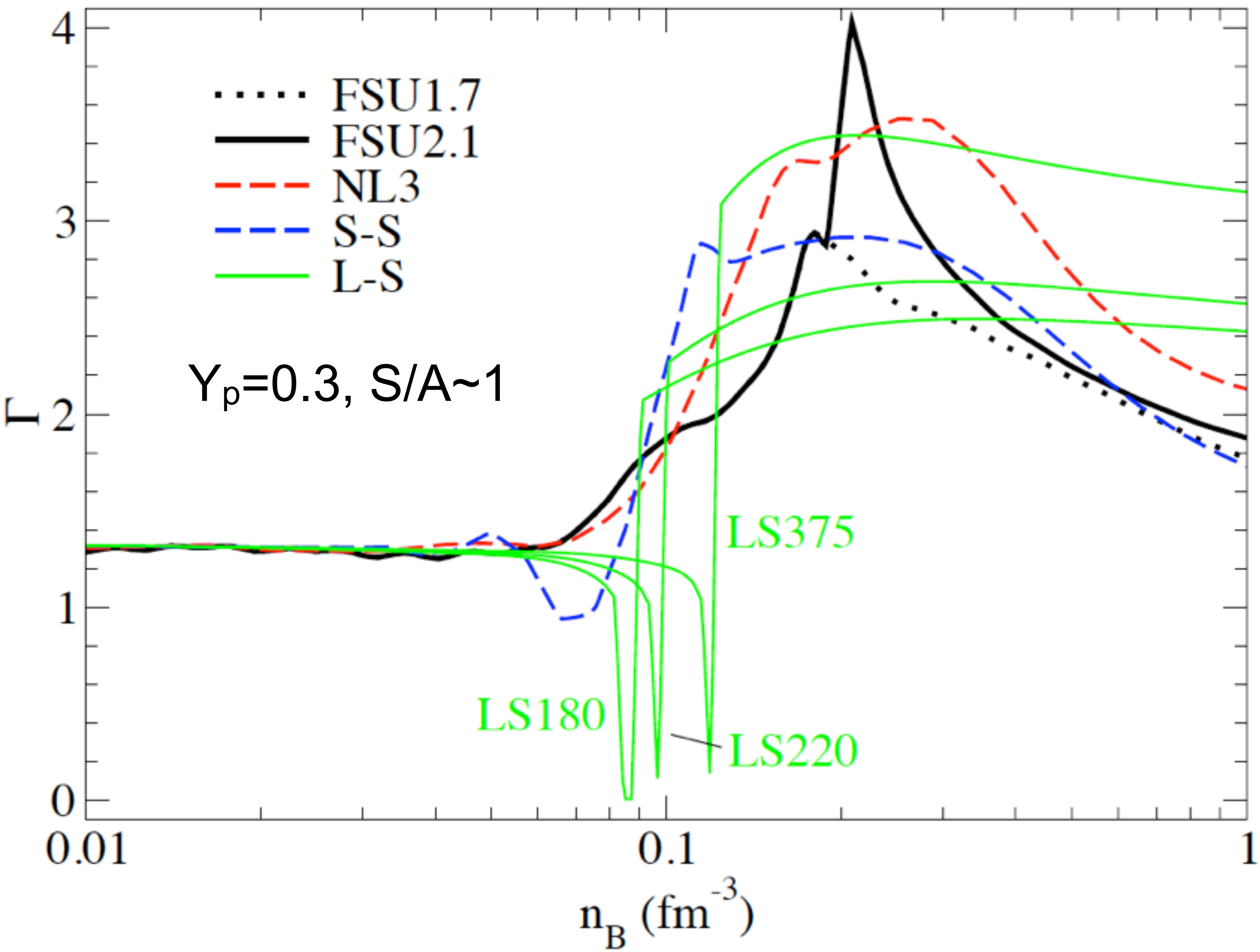
Thermodynamic Consistency

- Numerical noise and interpolation errors can lead to violation of 1st law etc.
- Start with (slightly noisy) entropy S calculated on 180,000 point grid of n_B, T, Y_p.
  - Numerically smooth S subject to $dS/dT>0$, $dS/dn<0$.
  - Interpolate S to fine ~2,000,000 point grid. Integrate S wrt T to get free energy on fine grid.
  - Differentiate this free E to get P, chemical potentials...
- Full EOS tables are available: http://cecelia.physics.indiana.edu/gang_shen_eos

  [G. Shen et al, arXiv:1103.5174]

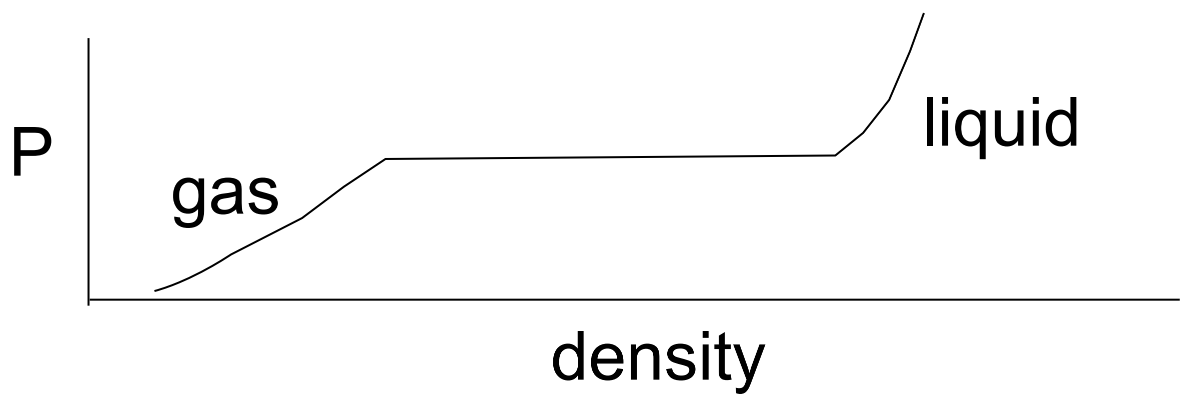
Skeleton in closet

Adiabatic Index $\Gamma = d\ln P / d\ln n_B$

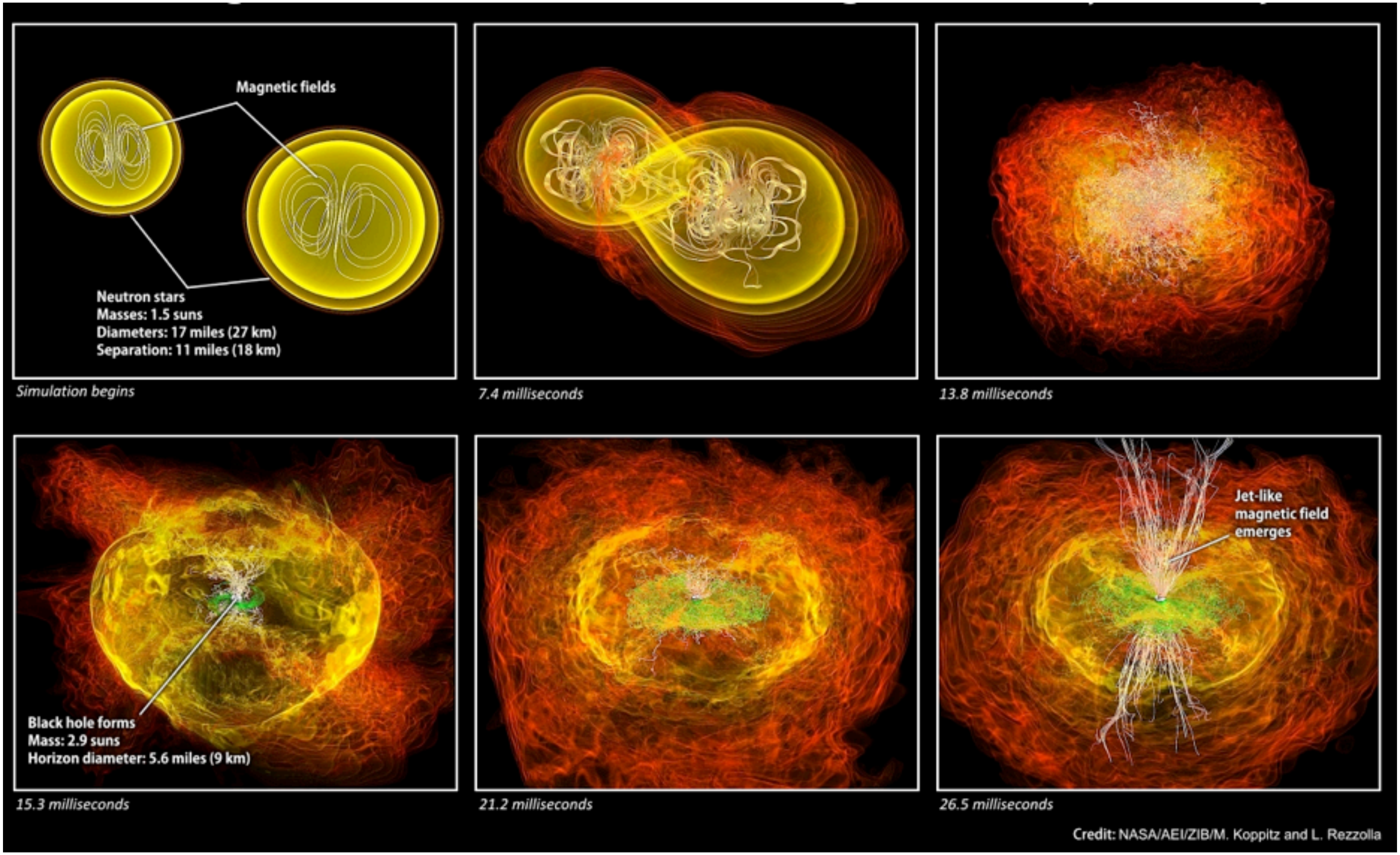


Liquid-Gas Phase Transition

- In heavy ion collisions, coulomb is not dominant, and one can have a 1st order phase transition.
- In astrophysics, coulomb implies average proton density = electron density. Fraction of system that is liquid is fixed. ***NO large region with P independent of density.***
- 1st order phase transition for LS EOS is artifact of simple approximations.



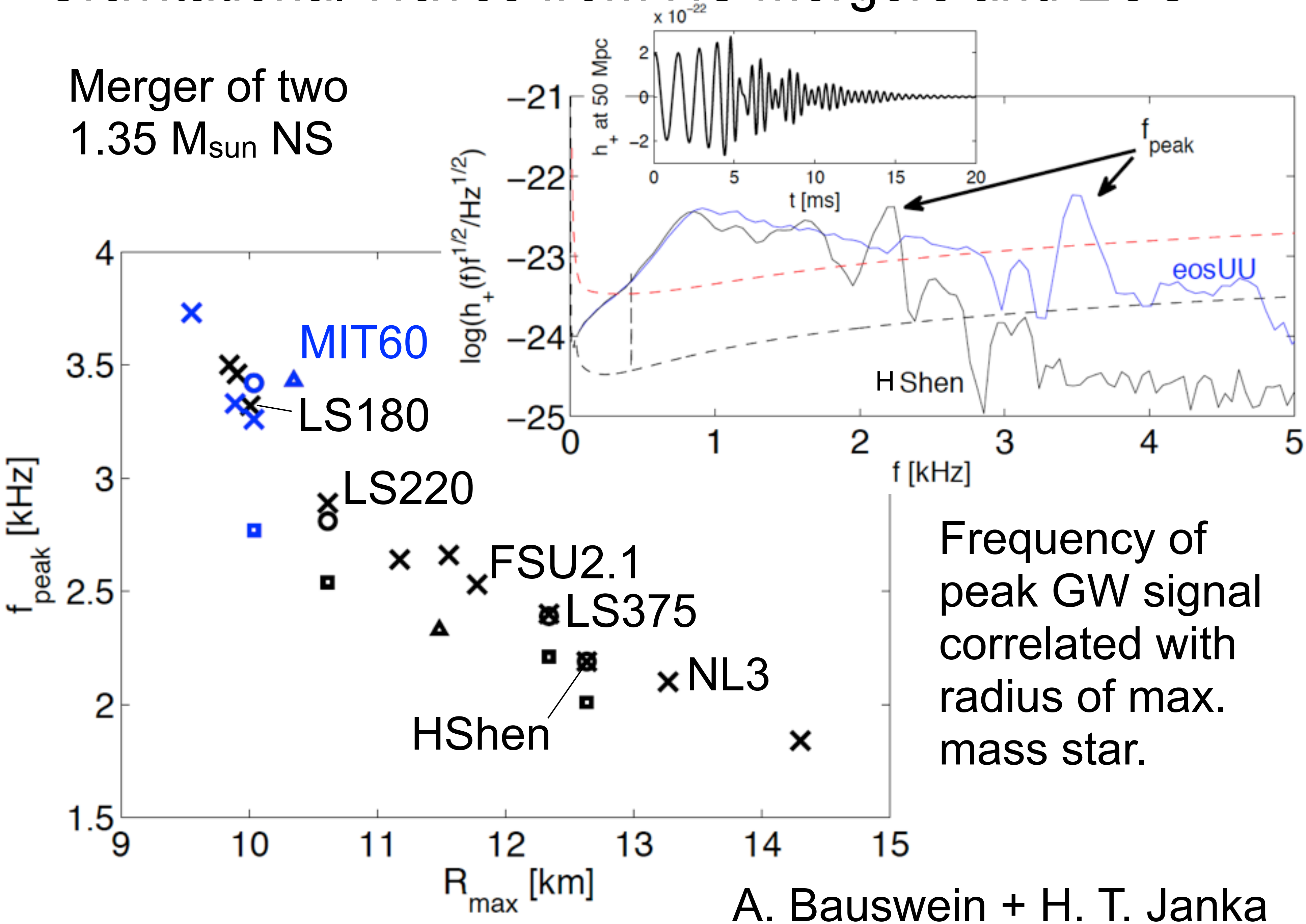
Neutron star mergers and gravitational waves



We anticipate the historic detection of GW within a few years with the operation of Advanced LIGO,and VIGRO.

Gravitational Waves from NS mergers and EOS

Merger of two 1.35 M_{sun} NS



Frequency of peak GW signal correlated with radius of max. mass star.

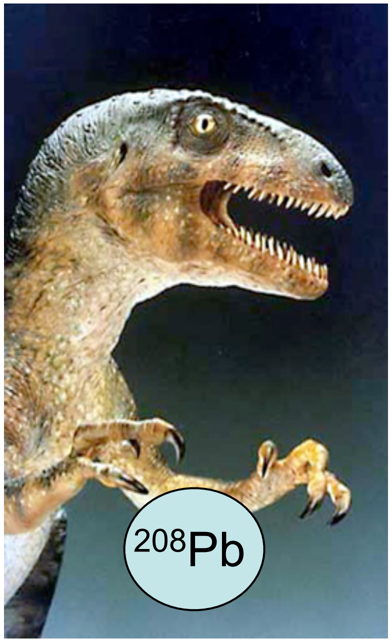
A. Bauswein + H. T. Janka

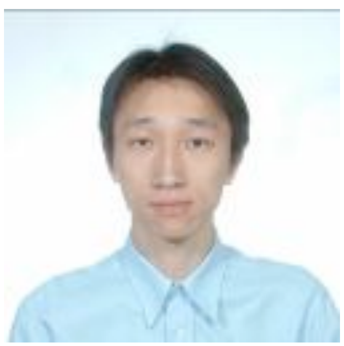
Equation of State Table

- ~2 million lines with 16 numbers per line. Each line has:
  - 9 thermodynamic quantities: T, Y_p, n, F, P, S, μ_n, μ_p, μ_e
  - 7 quantities provide composition information for neutrino interactions: x_n, x_p, x_a, x_h, <A>, <Z>, M* (mass fractions for neutrons, protons, alphas, and heavy nuclei, average A, Z of heavy nucleus, effective mass)
- Mass fraction of alpha particles, etc., only has meaning in association with routine to calculate neutrino interactions.

Skeleton in closet

Astrophysical Equations of State





Gang Shen

- PREX uses parity violating electron scattering to accurately measure the neutron radius.
  - First result: R_n-R_p(^{208}Pb)=$0.34^{+.15}_{-.17}$ fm.
  - Many implications for astrophysics...
- New equations of state use virial/ statistical model at low densities and extensive relativistic mean calculations at high densities.
- Supernova nu-sphere described by model independent Virial expansion. *Neutrino response enhanced by clustering.*
- Collaborators: A. Schwenk, E. O'Connor, J. Piekarewicz ... Students: L. Caballero, H. Dussan, J. Hughto, A. Schneider, and *G. Shen (did lions share of EOS work).*
- Supported in part by DOE.