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Treatment of HFB Resonances

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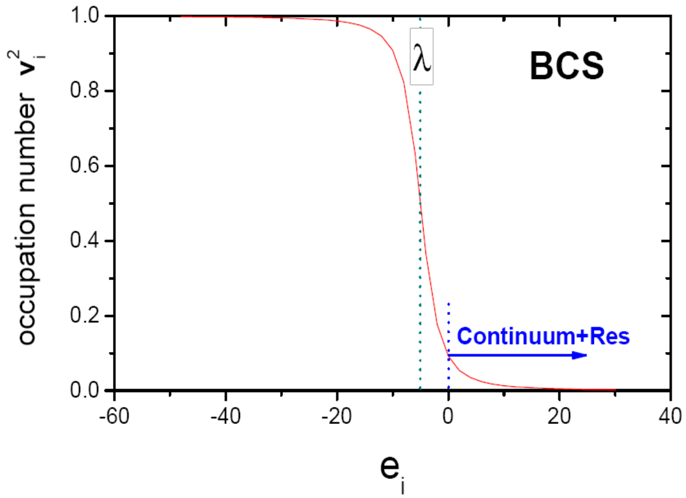
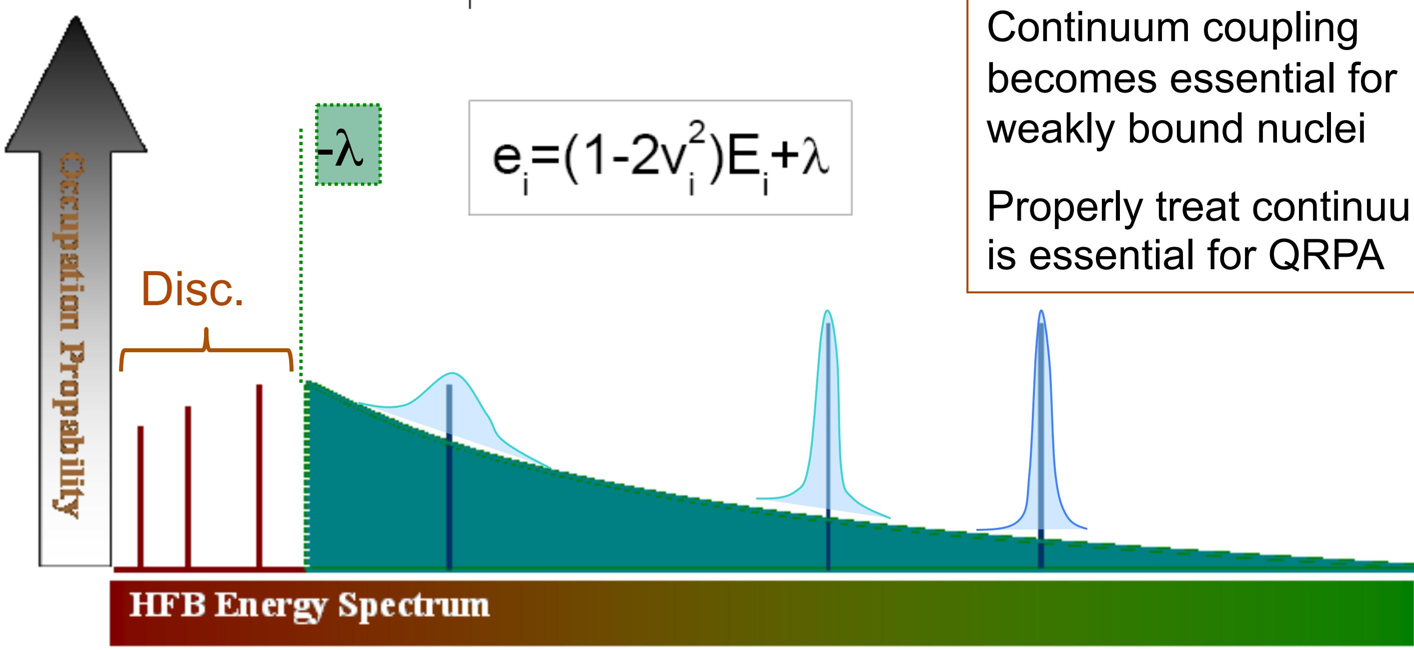
HFB Quasiparticle spectrum

- BCS



- HFB



$$e_i=(1-2v_i^2)E_i+\lambda$$

Occupation numbers

$$v_k^2=\frac{1}{2}\left(1-\frac{\tilde{\varepsilon}_k}{\sqrt{\tilde{\varepsilon}_k^2+\Delta_k^2}}\right)$$

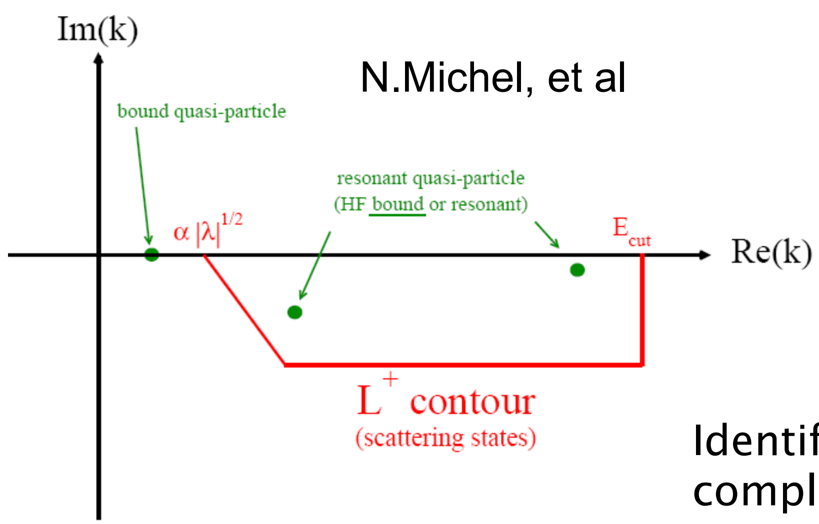
HFB deep-hole states become resonances

Continuum coupling becomes essential for weakly bound nuclei

Properly treat continuum is essential for QRPA

HFB Quasiparticle spectrum

- In the complex k plane



N.Michel, et al

Identify resonance states that have the complex energy: E-iΓ/2 (widths); and purely scattering states are integrated over the contour.

- These resonances are observable in experiments; widths depend very much on the pairing interaction; unique in HFB theory beyond BCS description

HFB Strategy

- Diagonalization on single-particle basis
- Direct diagonalization on box lattice (e.g. B-spline)

  Very expensive; so many discretized continuum states
- Madness-HFB strategy:

  –> initial HO basis
  –> Hamiltonian matrix
  –> diagonalization
  –> new eigenvalues and vectors
  –> Convolution(update basis)
  –> box boundary(mask)

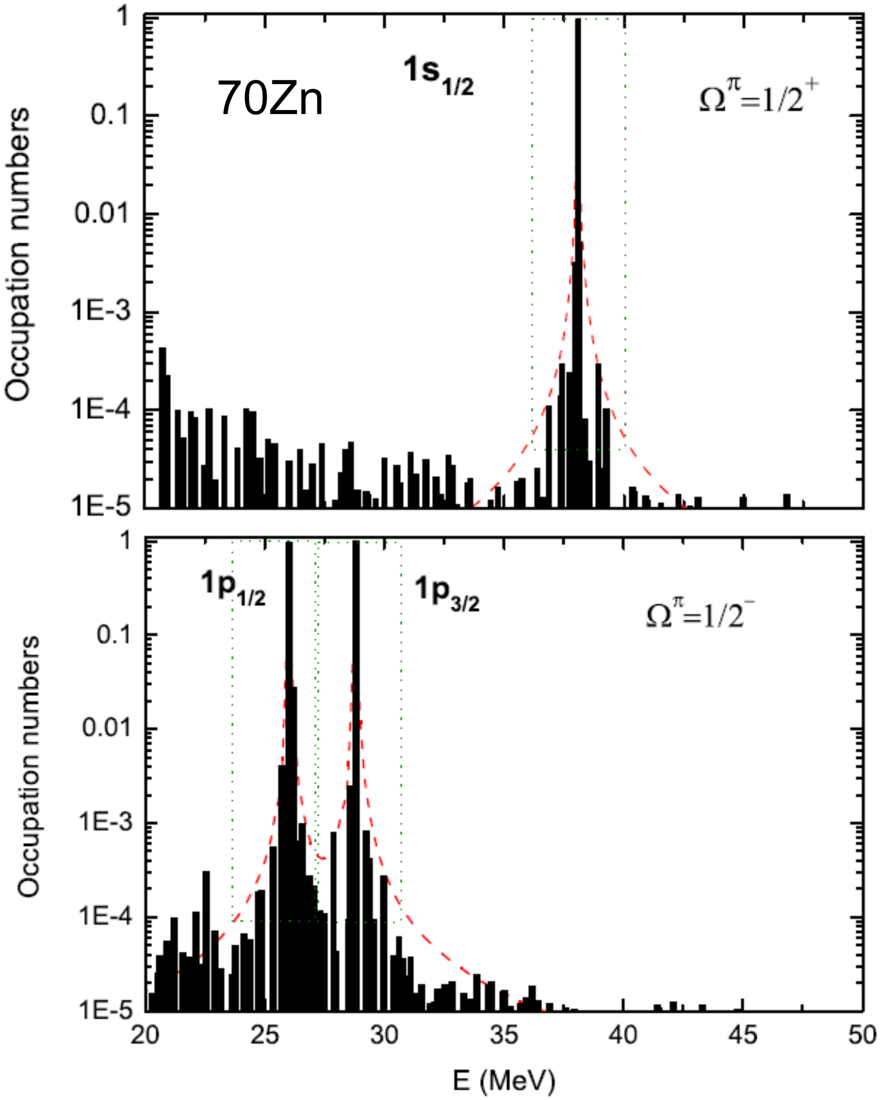
Hybrid-HFB strategy:

box solutions+deep-hole resonances+high energy continuum states

Box solutions

- In the box solution, also called *L^2* discretization, the continuum is discretized into finite states, with very good accuracy compared to exact treatments.
- Even with this method the resonance widths can be calculated precisely, compared to the complex scaling method.
- HFB-AX generates very dense quasiparticle spectrum, provides a high resolution for continuum and resonance states.

  For example, about 7000 states in a 40×40 fm box.



High energy continuum states

- High-energy approximation (local density approximation)

$$u(\boldsymbol{r}) \rightarrow u(\boldsymbol{p},\boldsymbol{r})e^{i\hbar\phi(\boldsymbol{r})}, \quad v(\boldsymbol{r}) \rightarrow v(\boldsymbol{p},\boldsymbol{r})e^{i\hbar\phi(\boldsymbol{r})}$$

- This approximation works for HFB-popov equation for Bose gas;

  works for Bogoliubov de Gennes equations for Fermi gas.

  J. Reidl, A. Csordas, R. Graham, and P.Szepfalusy, Phys. Rev. A 59, 3816 (1999)

  X.J. Liu, H. Hu, P.D. Drummond, Phys. Rev. A 76,043605 (2007)

- We follow this approximation for Skyrme HFB.

$$\left(\frac{\hbar^2 p^2}{2M^*} + V(\boldsymbol{r}) - \lambda\right)u(\boldsymbol{p},\boldsymbol{r}) + \Delta(\boldsymbol{r})v(\boldsymbol{p},\boldsymbol{r}) = \epsilon(\boldsymbol{p},\boldsymbol{r})u(\boldsymbol{p},\boldsymbol{r})$$

$$\left(\frac{\hbar^2 p^2}{2M^*} + V(\boldsymbol{r}) - \lambda\right)v(\boldsymbol{p},\boldsymbol{r}) - \Delta(\boldsymbol{r})u(\boldsymbol{p},\boldsymbol{r}) = -\epsilon(\boldsymbol{p},\boldsymbol{r})v(\boldsymbol{p},\boldsymbol{r})$$

$$\epsilon(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\varepsilon^2_{\mathrm{HF}}(\boldsymbol{p},\boldsymbol{r}) + \Delta(\boldsymbol{r})^2}$$ quasiparticle spectrum

$$\varepsilon_{\mathrm{HF}}(\boldsymbol{p},\boldsymbol{r}) = \frac{\hbar^2 \boldsymbol{p}^2}{2M^*(\boldsymbol{r})} + V(\boldsymbol{r}) - \lambda$$ HF energy

Derivatives of effective mass, spin-orbit terms omitted for high energy states

Continuum to observables

$$\rho_c(\boldsymbol{r}) = \sum_p [1 - \frac{\varepsilon_{\text{HF}}}{\epsilon}]\Theta[\epsilon - E_c],$$

$$\rho_c(\boldsymbol{r}) = \frac{M^*}{2\pi^2\hbar^2}\int_{\epsilon_c} d\epsilon(\frac{\epsilon}{\sqrt{\epsilon^2-\Delta^2}} - 1)\times \sqrt{\frac{2M^*}{\hbar^2}(\sqrt{\epsilon^2-\Delta^2} - V(\boldsymbol{r}) + \lambda)},$$

$$\tau_c(\boldsymbol{r}) = \sum_p p^2[1 - \frac{\varepsilon_{\text{HF}}}{\epsilon}]\Theta[\epsilon - E_c]$$

$$\tau_c(\boldsymbol{r}) = \frac{M^*}{2\pi^2\hbar^2}\int_{\epsilon_c} d\epsilon(\frac{\epsilon}{\sqrt{\epsilon^2-\Delta^2}} - 1)\times [\frac{2M^*}{\hbar^2}(\sqrt{\epsilon^2-\Delta^2} - V(\boldsymbol{r}) + \lambda)]^{3/2}$$

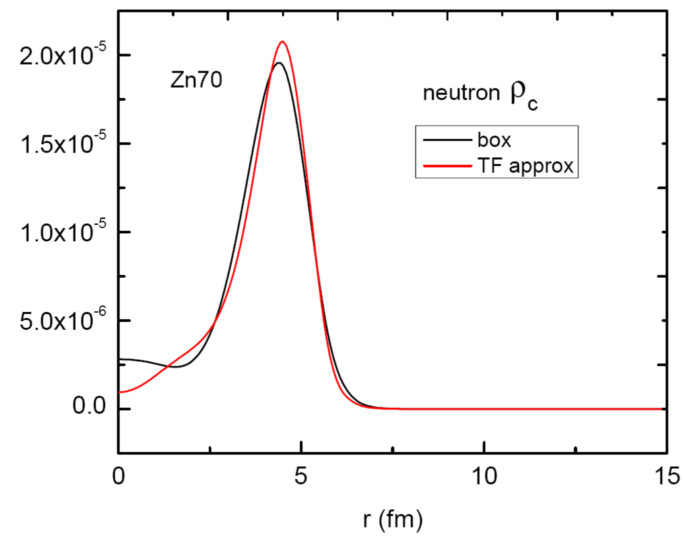
$$\tilde{\rho}_c(\boldsymbol{r}) = -\sum_p v_p u_p = -\sum_p \frac{\Delta}{\epsilon}\Theta[\epsilon - E_c]$$

$$\tilde{\rho}_c(\boldsymbol{r}) = -\frac{M^*\Delta}{2\pi^2\hbar^2}\times \int_{\epsilon_c} d\epsilon \frac{\sqrt{\frac{2M^*}{\hbar^2}(\sqrt{\epsilon^2-\Delta^2} - V(\boldsymbol{r}) + \lambda)}}{\sqrt{\epsilon^2-\Delta^2}}.$$

Transform from the *p*-representations to the integral in terms of energy

Continuum to observables

- Contributions from 40 to 60 MeV
  - Local density approximation
  - Box solution from HFB-AX

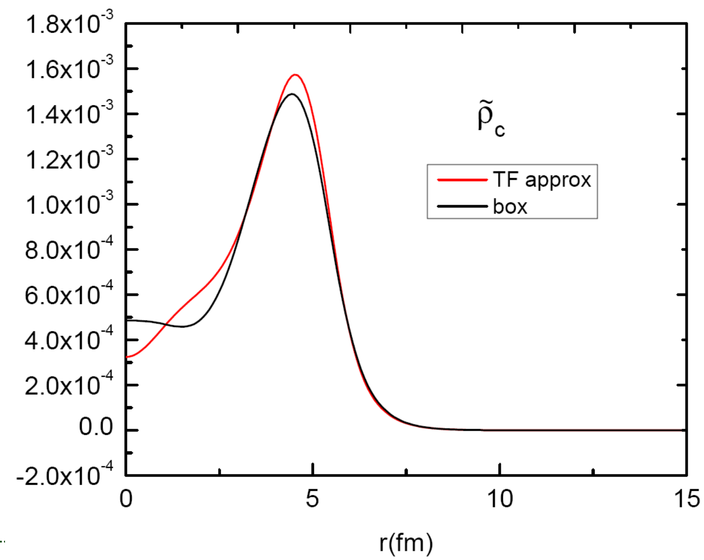


What we see:

Generally the two methods agrees with each other;

The distributions are very similar for the three kind of densities, and this depends on the pairing potential;

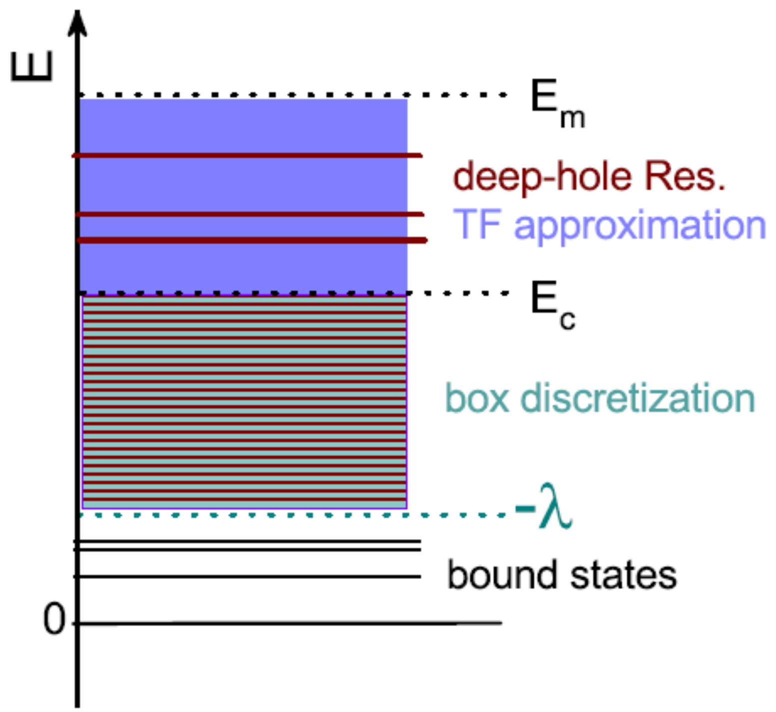
Continuum significantly impacts the *pp* channel.



Hybrid HFB

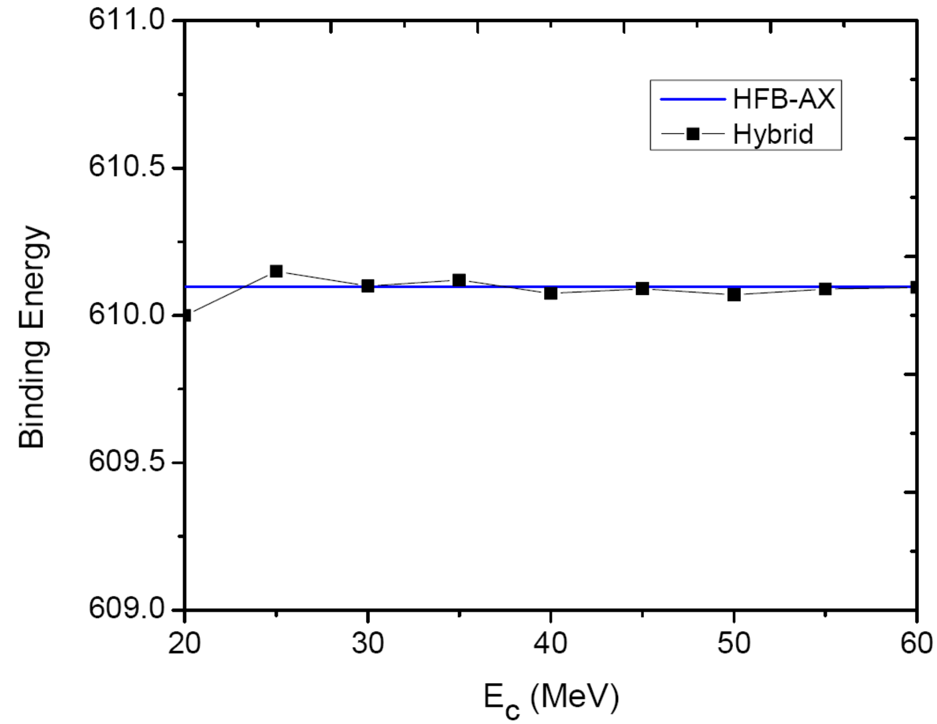
- Test the local-density approximation

Assume the continuum coupling is very weak for deep-hole states



$$\rho(\boldsymbol{r}) = \sum_{E_i>0}^{E_i<Ec1} 2|v_i|^2 + \sum_{\text{deep-hole}} 2|v_i|^2 + \rho_c(\boldsymbol{r}).$$

Test for Zn70 neutrons



Similar to pairing regularization method

HFB resonances

General formulas for resonance widths:

- Continuum level density

$$\Delta(E) = Tr[\delta(E-H) - \delta(E-H_0)]$$

Hamilton without interaction

With the Green function method:

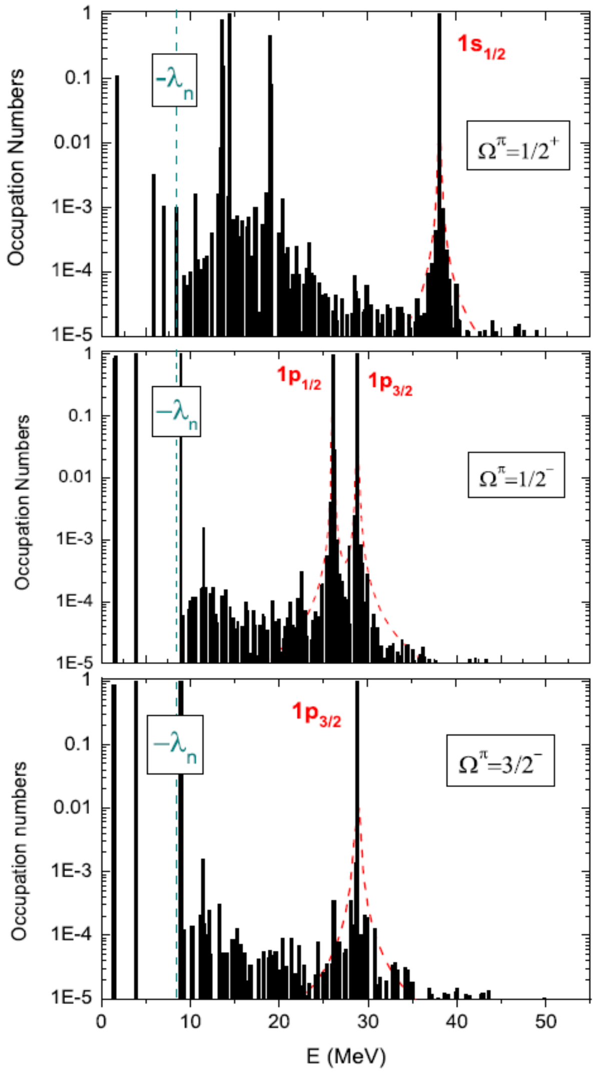
$$\Delta_l(E) = -\frac{1}{\pi}\,\mathrm{Im}\{\mathrm{Tr}[\hat{G}_l(E+i0) - \hat{G}_l^0(E+i0)]\}$$

Breit-Wigner shape for resonances

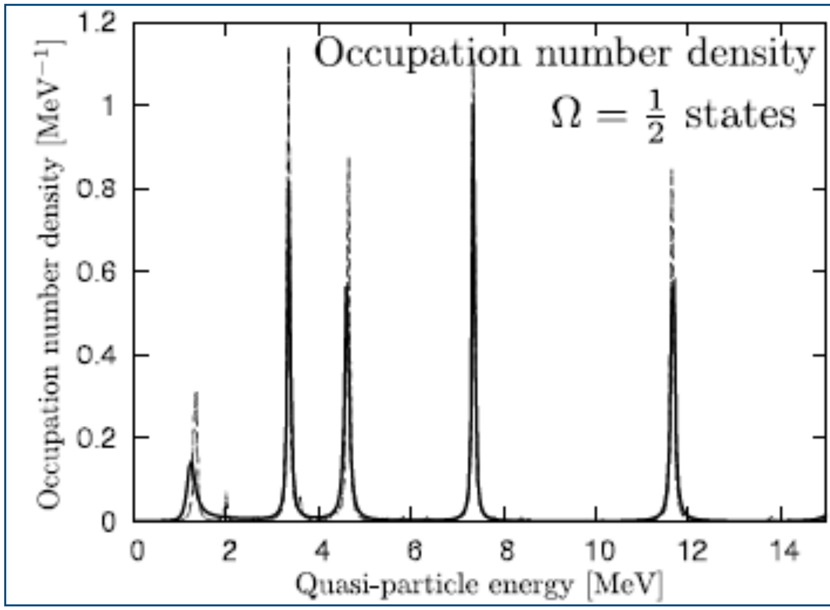
$$\Delta_l^r(E,E_r,\Gamma_r) = \frac{1}{\pi}\frac{\Gamma_r/2}{(E-E_r)^2+\Gamma_r^2/4}$$

Smoothing-Fitting method

- Occupation probability and continuum level density



$$n(E) = \frac{1}{\pi}\mathrm{Im}\sum_{\sigma}\int dr G_{11}(r\sigma, r\sigma, -E-i\epsilon)$$



*H. Oba, M. Matsuo, PRC **80**, 02430, 2009*

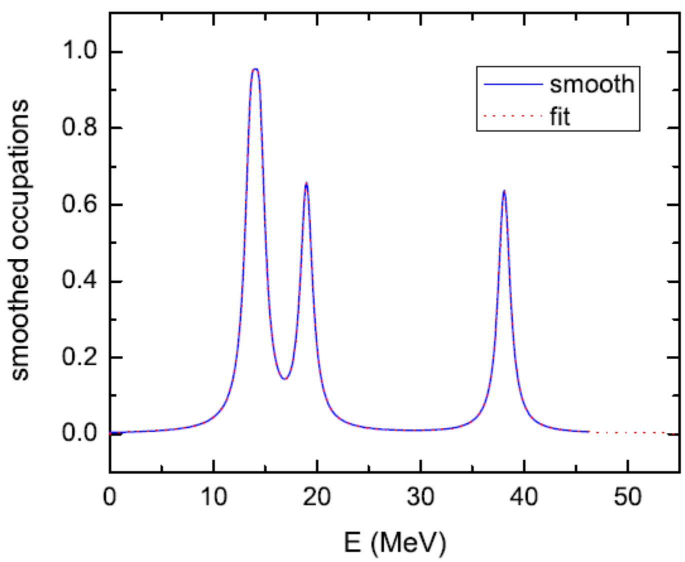
Occupation probability is related to the continuum level density, which corresponds to the Breit-Wigner shape

Smoothing-fitting method

- Straightforward smoothing-fitting method to extract the widths
  - Strutinsky smoothing with a Lorentz function

$$\overline{v^2}(E) = \sum_i \frac{v_i^2}{\Gamma} w\left(\frac{E - E_i}{\Gamma}\right)$$

A.T. Kruppa, K. Arai, PRA 59,3556 (1999).



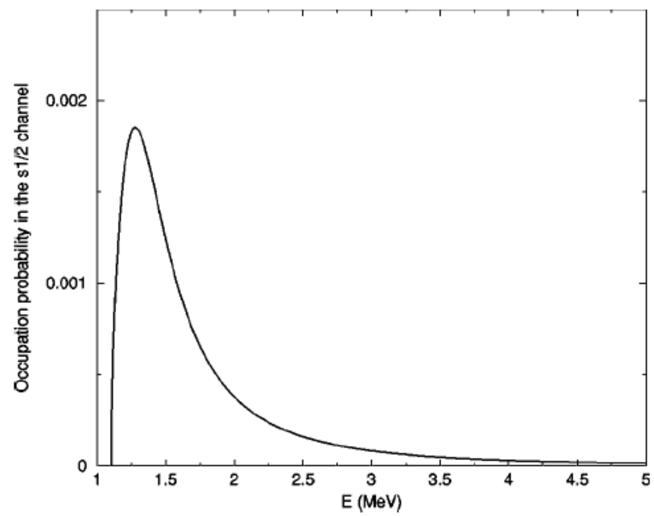
Advantages: Works for resonances in deformed nuclei; precision can be improved using a large box size

  - Then we do fitting:

$$\overline{v^2}(E) = v^2 \overline{v_R^2}(E) + v_b^2(E)$$

Total occupation

Smoothed B-W

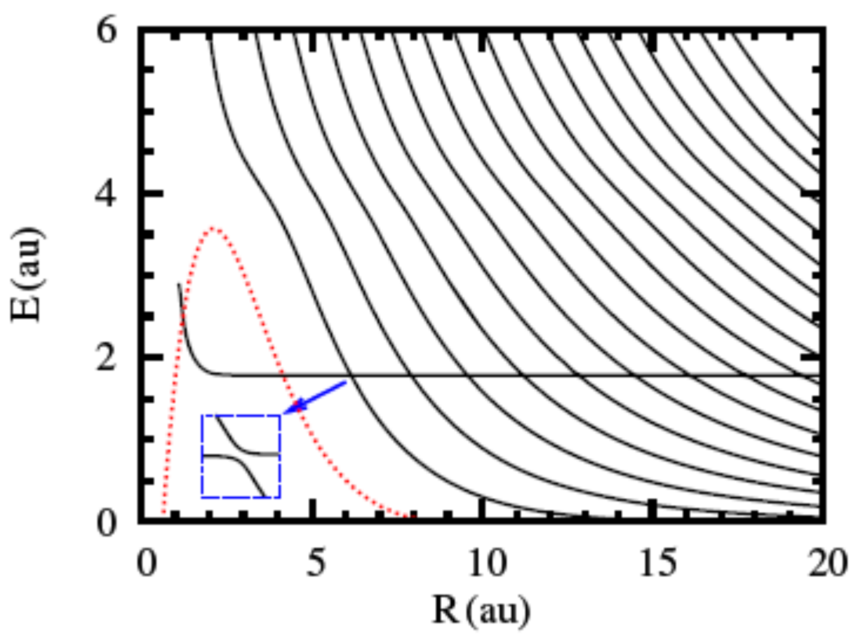


However, resonances near thresholds do not have a B-W shape

M. Grasso,PRC 64, 064321, 2001

Stabilization Method

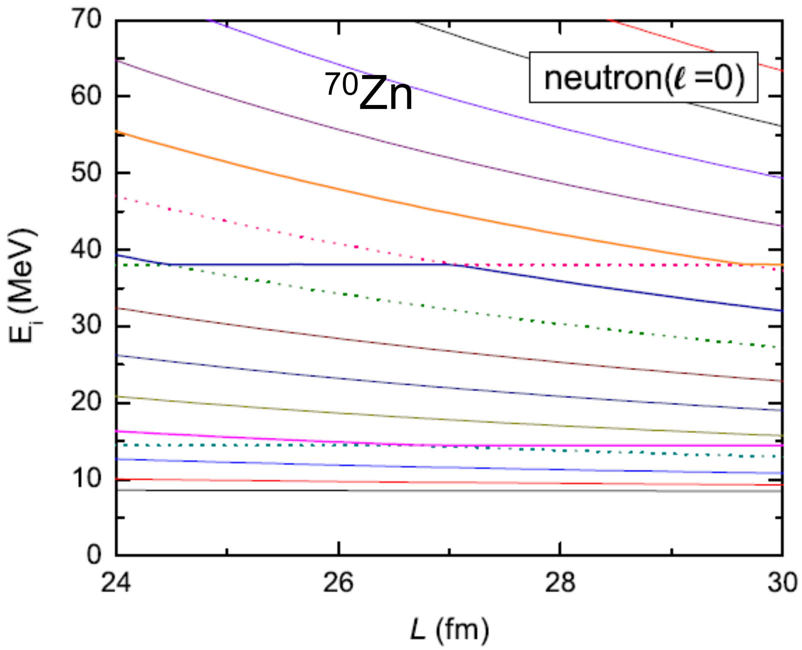
- Widely used in atomic physics
- Has been applied to single particle resonances
- 3-body resonances (A.Kruppa)
- We applied for the first time to quasiparticle resonances



S.G. Zhou, J. Meng, P. Ring, 2009

Stabilization method

- The main idea is to get resonance widths from the box-size dependence of quasiparticle energies



Spectrum got by HFBRAD solutions with different box sizes

Phase shift: $\eta(E) = \pi \int_0^E \Delta(E')dE'$

Continuum level density: $\Delta(E) = Tr[\delta(E-H) - \delta(E-H_0)]$

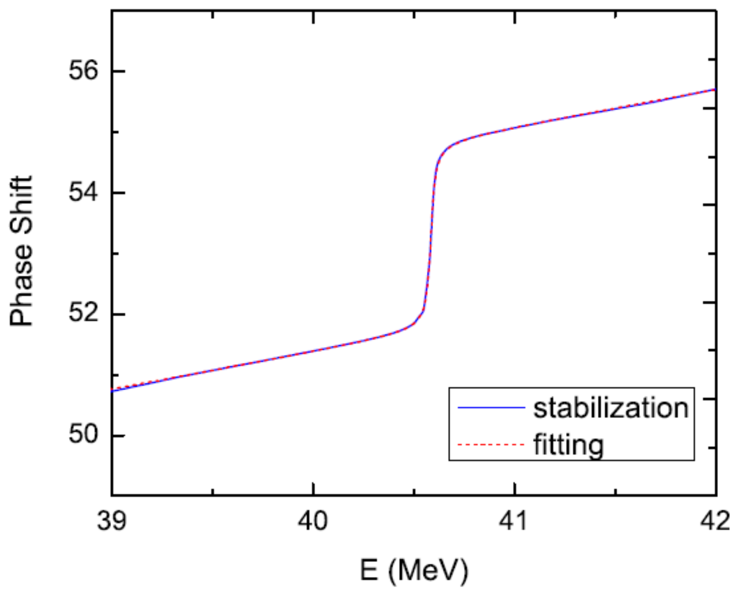
V.A. Mandelshtam, H.S. Taylor, V. Ryaboy and N. Moiseyev, PRA 50, 2764 (1994)

Stabilization method

- Calculate the phase shift

$$L = L_0 + \Delta L, \quad \Delta L = (M-1)\delta L$$

$$\eta(E) = \pi N(E) + \frac{\pi}{\Delta L}\sum_j (L_0 + \Delta L - L_j(E))$$

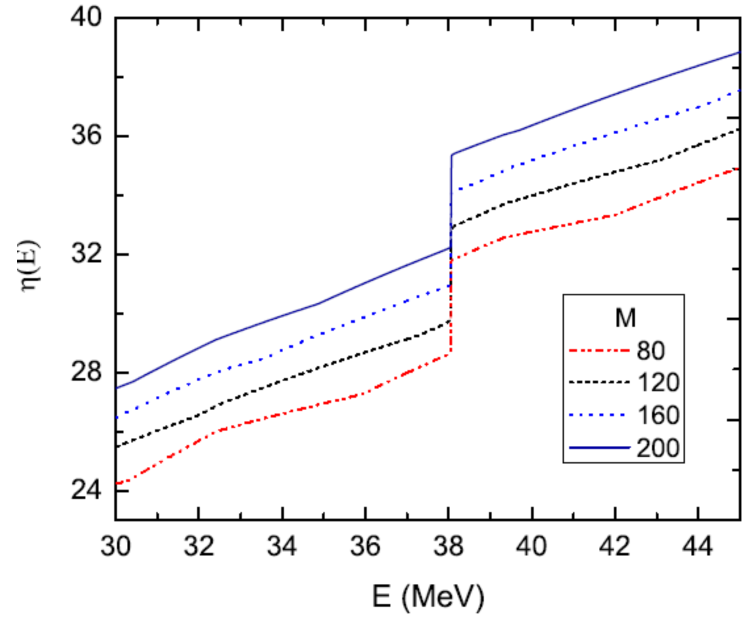


- Fitting with resonance phase shift and background

$$\varphi(E) = \arctan\left(\frac{2(E-E_r)}{\Gamma_r}\right) + \varphi_b(E)$$

Precision depends on two things:

The step size and the step numbers ***M***



$$\delta L \to 0 \quad \text{and} \quad \Delta L \to \infty$$

In principle, we can get very accurate results, but it can be expensive

Fermi Golden rule

- Assume the paring coupling can be treated perturbatively

$$\Gamma = 2\pi| < u_{0E}|\Delta(r)|\psi_0| > |^2$$

$$[\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r})]\frac{\psi_0(r)}{r}Y_{lm} = (-E+\lambda)\frac{\psi_0(r)}{r}Y_{lm}$$

$$[\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r})]\frac{u_{0E}(r)}{r}Y_{lm} = (E+\lambda)\frac{u_{0E}(r)}{r}Y_{lm}$$

Solve the scattering wavefunction

$$u_{0E} = \sqrt{\frac{2m}{\hbar^2 \pi k}}(\cos\delta_l F_l(\eta, kr) + \sin\delta_l G_l(\eta, kr))$$

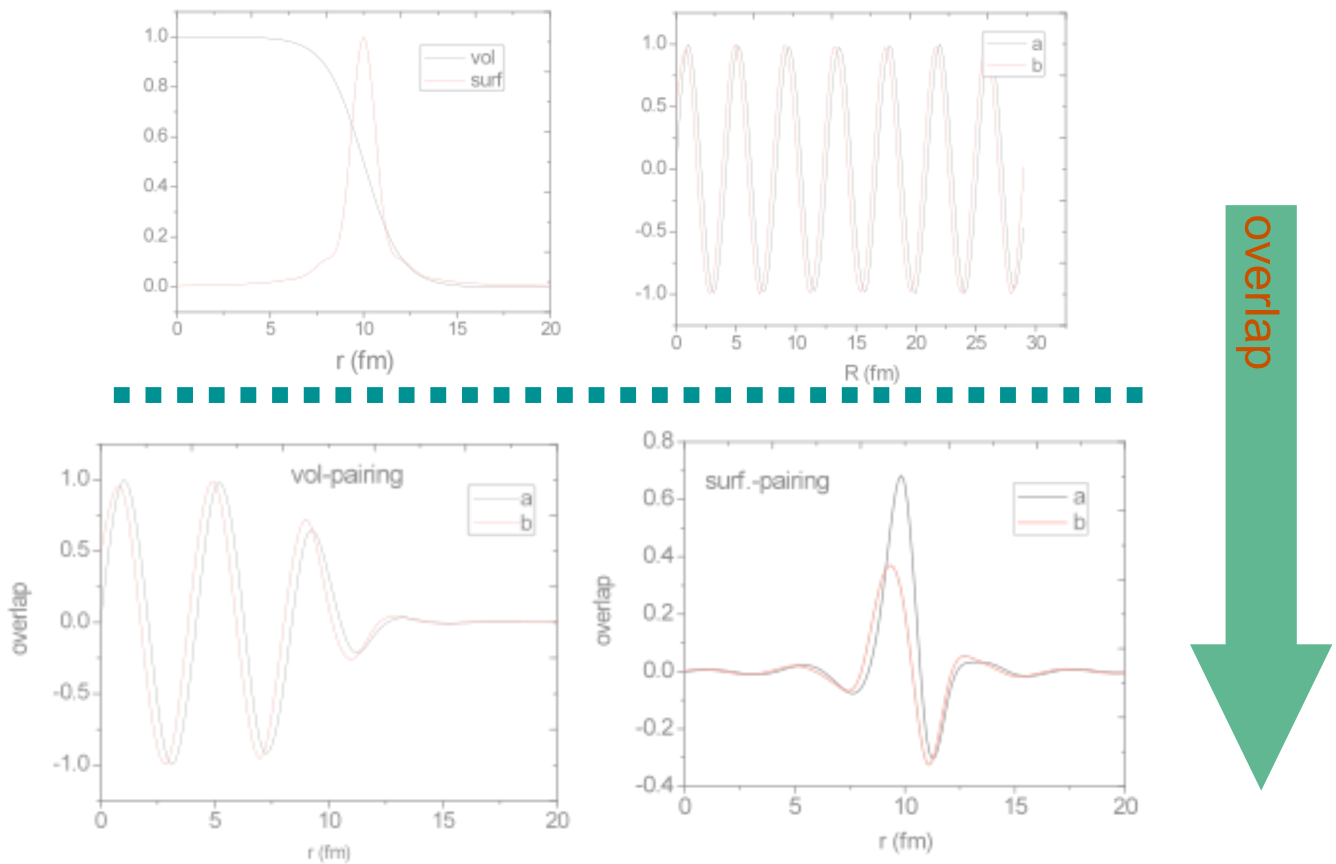
Asymptotic form at large distances

A. Bulgac, Preprint FT-194-1980; nucl-th/9907088

However, in some calculations, the FG rule is not precise.

Inapplicability of FG rule

- With the volume and surface pairing, two scattering functions *a, b*



- The overlap is too sensitive to the small changes in u_{0E} to give reliable widths, in the case of surface pairing.

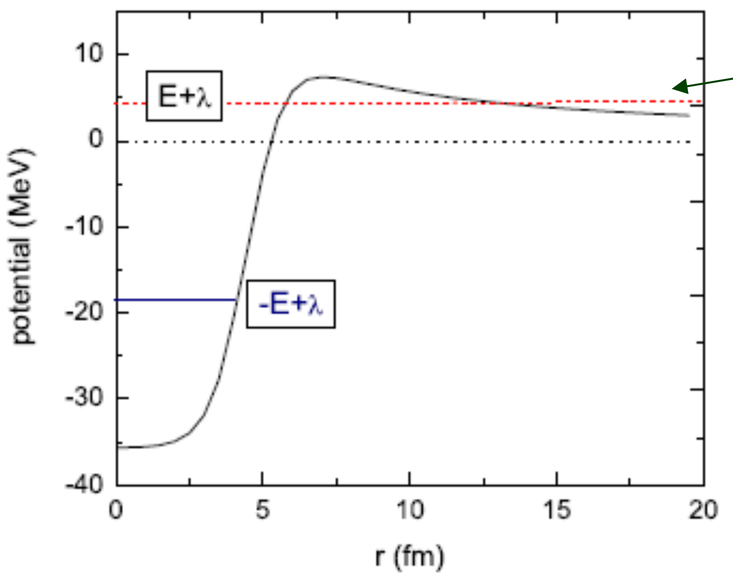
Width results

- comparison (widths: keV)

states	Neutron E	Neutron Γ_{box}	Neutron Γ_{smf}	Neutron Γ_{per}	Proton E	Proton Γ_{box}	Proton Γ_{smf}	Proton Γ_{per}
$1s_{1/2}$	38.075	1.23	0.80	0.98	35.147	0.64	0.54	0.40
$1p_{3/2}$	28.801	3.42	4.1	0.04	25.424	3.22	2.04	2.27
$1p_{1/2}$	26.043	6.01	7.0	0.19	22.851	2.04	1.57	1.26
$1d_{5/2}$	18.987	13.1	10.6	22.7	15.320	-	0.12	0.09

Fermi-golden rule doenst work

- Special narrow reonance ($1d_{5/2}$)



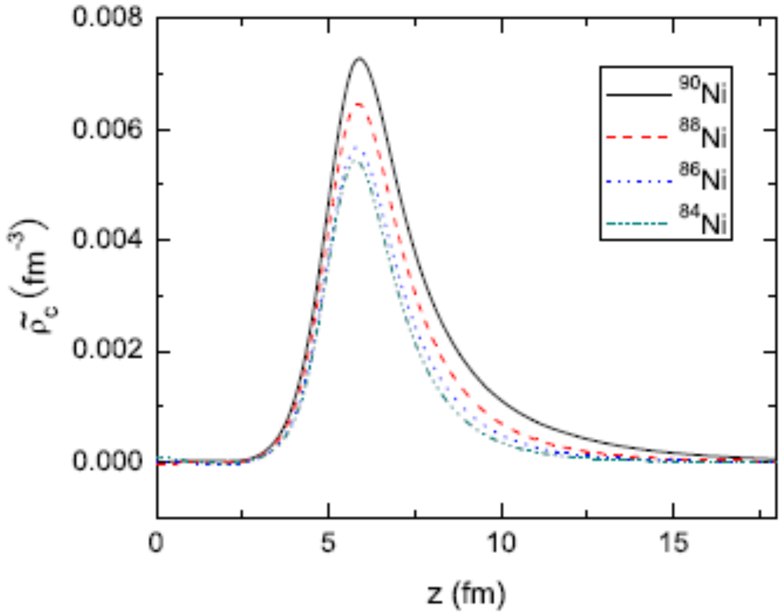
Resonance-like solution for the upper component

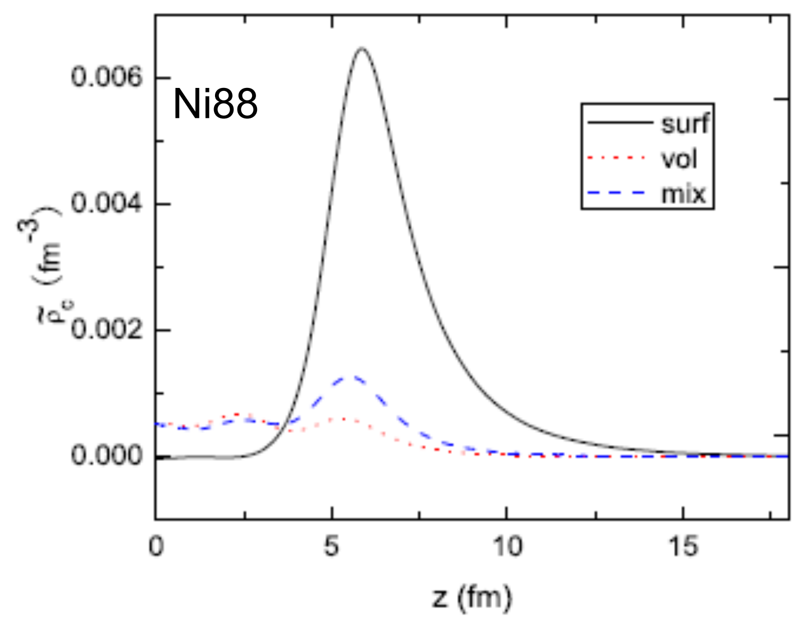
It is difficult for the stabilization method to calculate widths of extremely narrow resonances

Also happens in neutron states with centrifugal barrier

Weakly-bound Ni isotopes

- continuum contribution from 30 to 60 MeV





Continuum contribution increases as nuclei close to the drip line

Continuum contribution depends very much on the pairing interaction

Weakly-bound Ni isotopes

- Resonance widths (keV) of 90Ni

states	E_r	Γ_{box}	Γ_{smf}	Γ_{GA}[12]
$1s_{1/2}$	51.419	-	1.1e-3	1.09e-3
$1p_{3/2}$	40.588	30.84	20.17	27.28
$1p_{1/2}$	38.770	32.26	34.67	27.14
$1d_{5/2}$	29.039	1.31	1.37	0.78
$1d_{3/2}$	25.017	25.44	23.08	22.57
$2s_{1/2}$	24.319	50.36	40.87	46.00
$1f_{7/2}$	17.554	401.79	413.04	397.37
$2p_{3/2}$	12.538	499.98	471.22	490.56
$1f_{5/2}$	10.981	672.44	651.12	645.64
$2p_{1/2}$	10.816	440.76	376.19	404.30
$1g_{9/2}$	6.519	1.56	0.52	0.81
$2d_{5/2}$	3.270	221.08	105.17	194.18
$2d_{3/2}$	2.310	611.50	643.88	560.61
$1g_{7/2}$	3.348	69.09	75.13	63.61
$1h_{11/2}$	5.527	162.46	173.66	131.78

- For example, $1p_{3/2}$ in Ni isotopes: 25.9 keV (*N*=58), 28.2 keV (*N*=60), 30.8 keV (*N*=62)

Widths increase as nuclei close to the drip line

Summary

- local-density approximation:

  Works for high energy continuum states.Hybrid HFB strategy can greatly reduce computational costs; will be very useful for optimizing Madness-HFB. It also demonstrates that box solutions have very good precision, and can be used for continuum-QRPA.

- Stabilization method:

  Produce reliable widths except for very narrow resonances, slightly overestimate the widths by 10%. Provides an alternative way to study continuum and resonances without solving the scattering wavefuntions

- Drip-line isotopes:

  HFB resonance widths are increasing as nuclei close to drip-lines, which is consistent with that high-energy continuum contributions are also increasing.

Thanks for your attention!