

# TOWARDS UNEDF2 AND BEYOND

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## Collaboration

### UTK/ORNL

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### ANL

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### NSCL/MSU

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Heiko Hergert

### Ohio State University

Dick Furnstahl

### Iowa State University

Pieter Maris  
James Vary

### LANL

Joseph Carlson

### Warsaw & Jyväskylä University

Jacek Dobaczewski

### CEA

Thomas Duguet

### RIKEN

Takashi Nakatsukasa

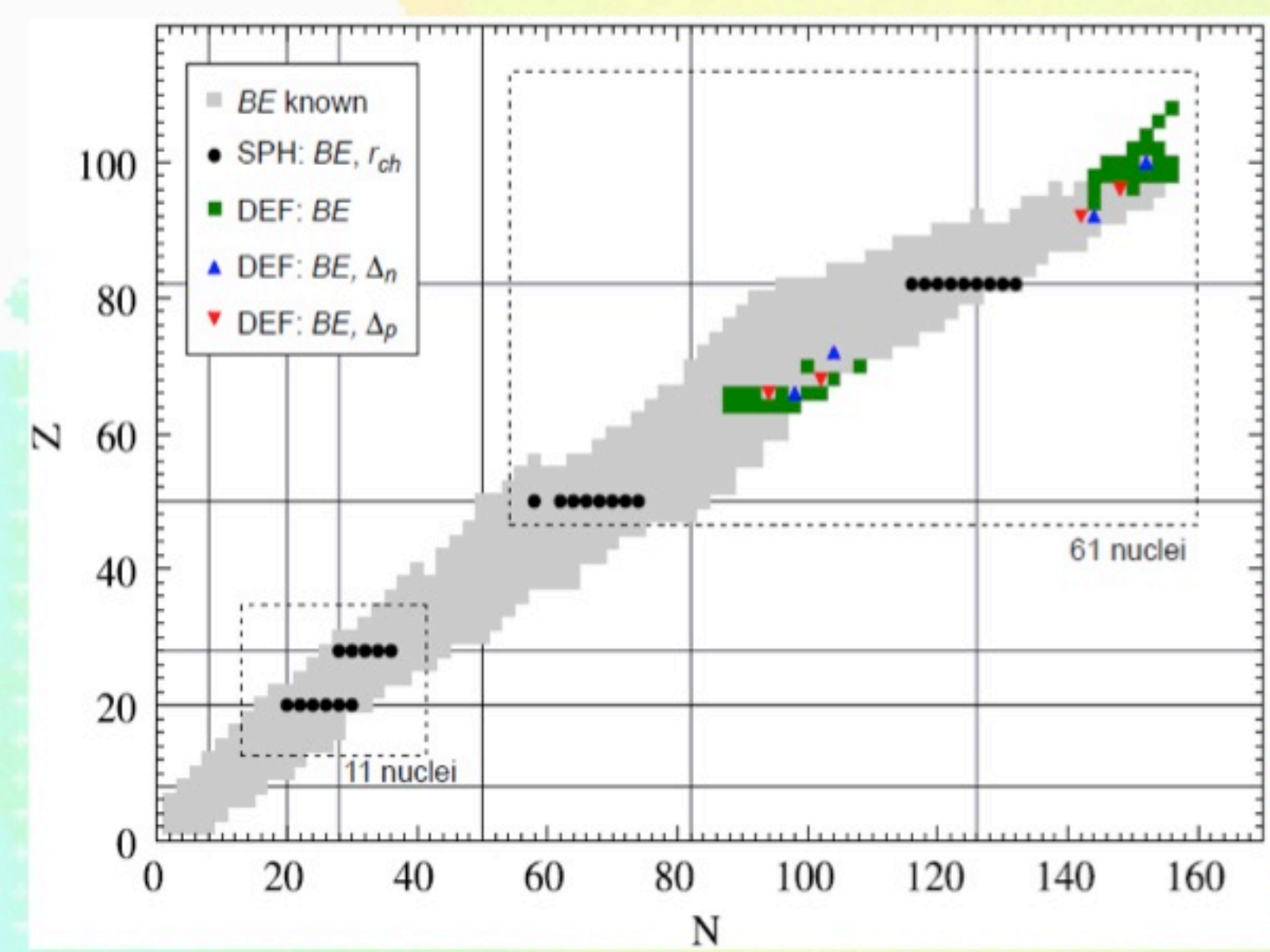




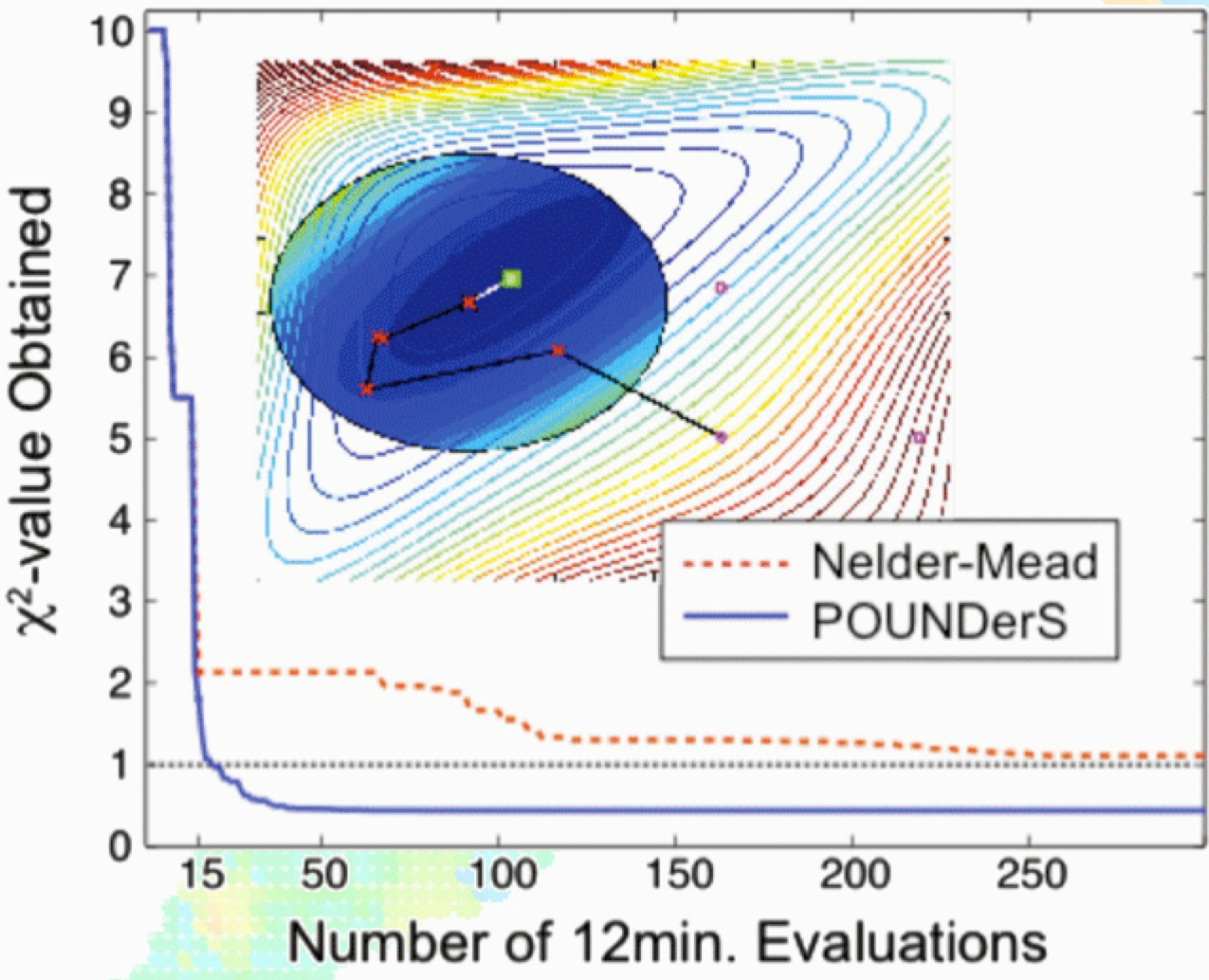
M.Kortelainen, T.Lesinski, J.Moré, W.Nazarewicz, J.Sarich, N.Schunck, M.V.Stoitsov, and S.Wild, Phys. Rev. C **82**, 024313 (2010)

- 12 parameters fit
- Emphasis on heavy nuclei ( $A > 100$ )

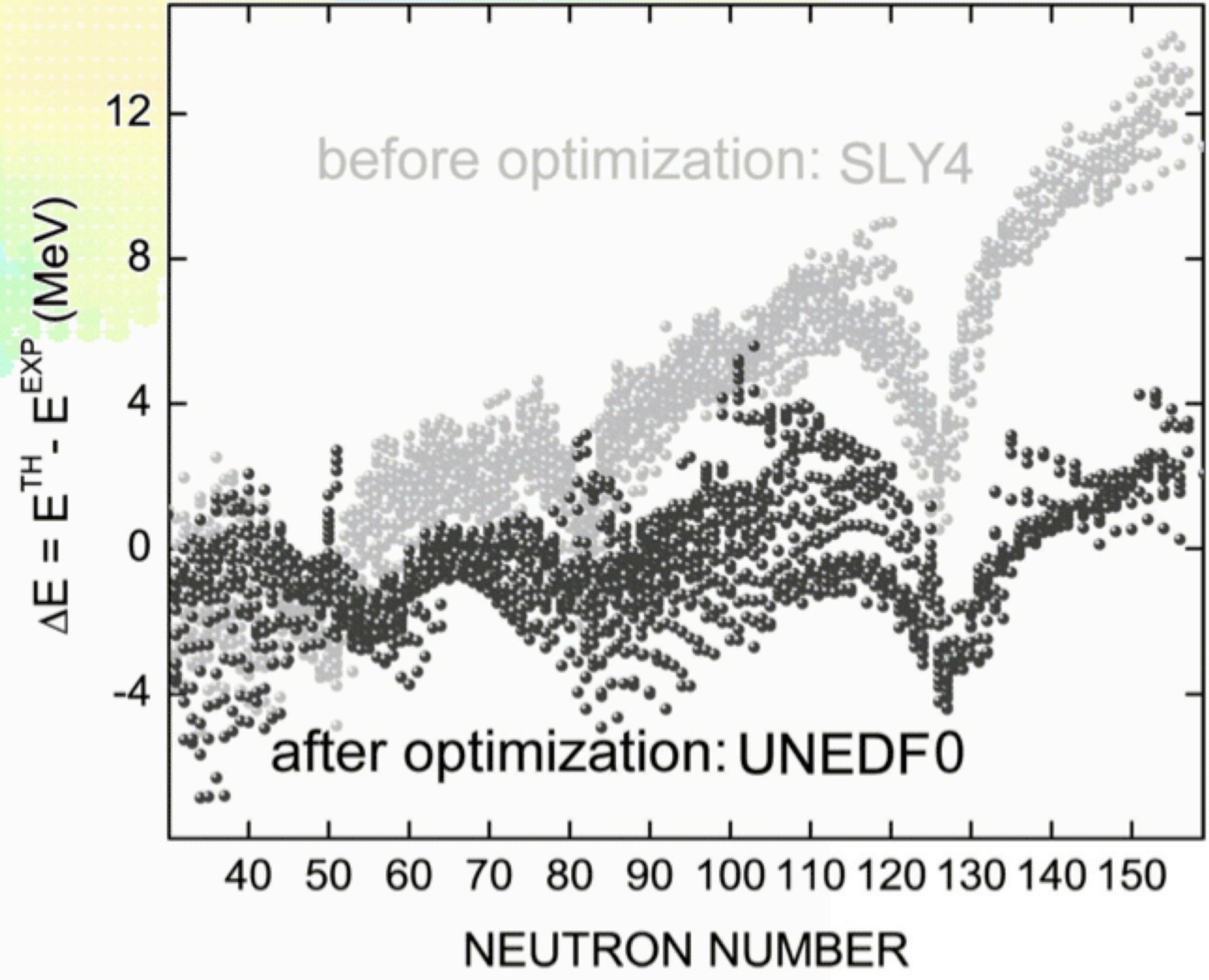
- 108 data points:
- 28 spherical masses
  - 28 r.m.s. radii
  - 44 deformed masses
  - 8 OEM points



Optimization of the energy functional



Calculating nuclear properties





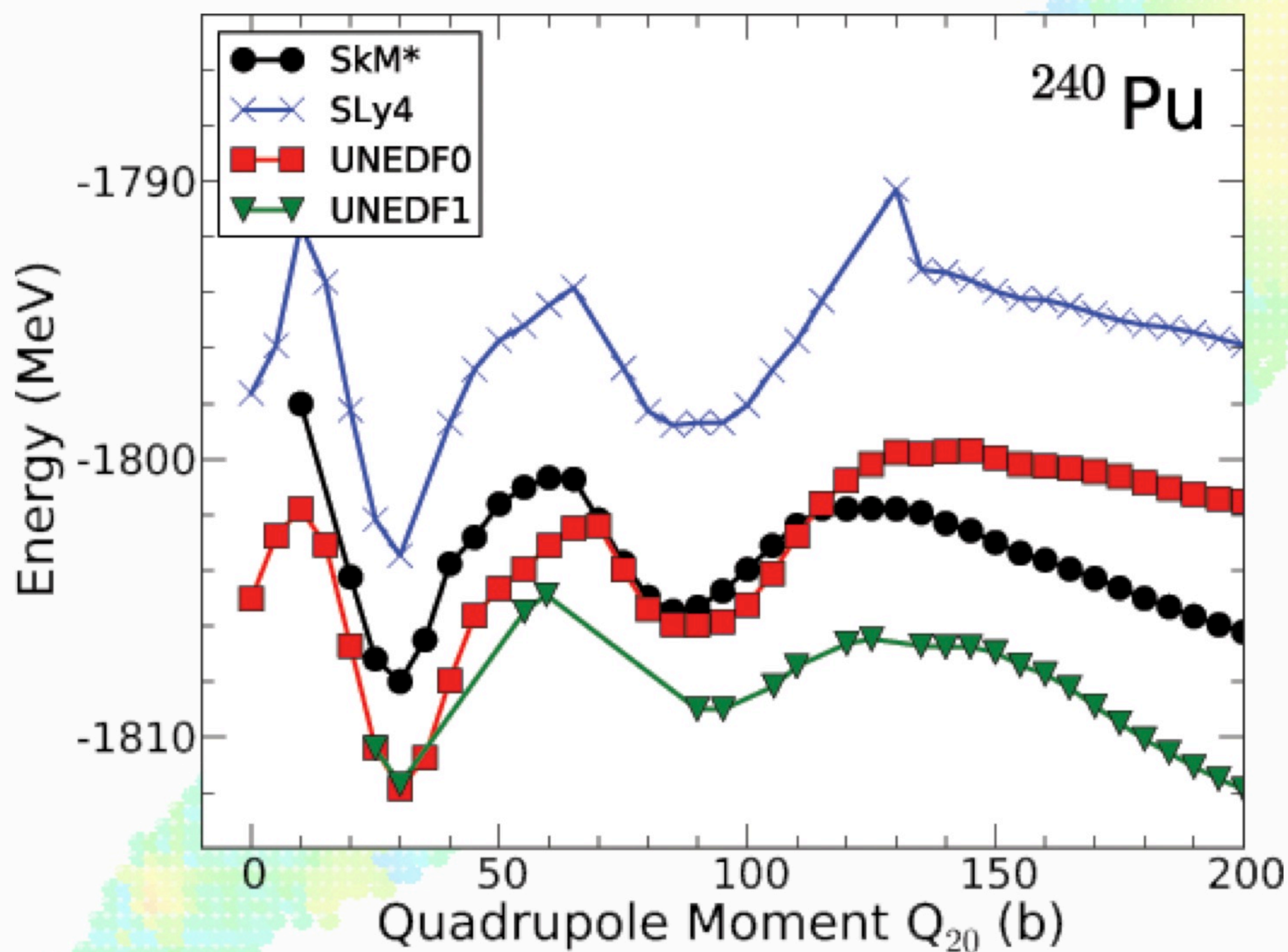
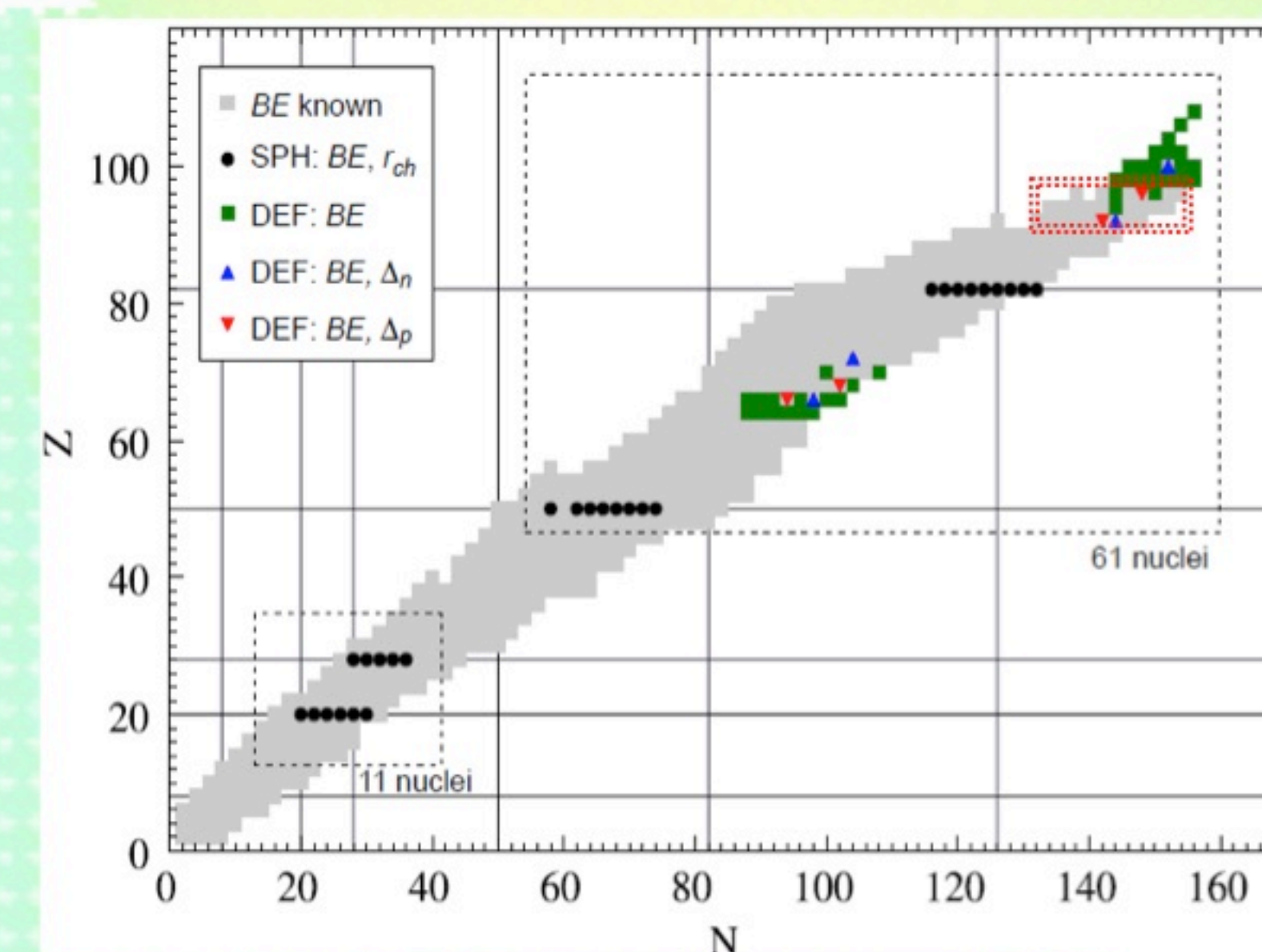
### New data points added

Z	N	E (MeV)	w (MeV)
92	146	-1800.98110	2.0
94	146	-1812.70387	2.0
96	146	-1822.56507	2.0

### Super deformed isomers added

Z	N	E (MeV)	w(MeV)
92	144	2.750	0.5
92	146	2.557	0.5
94	146	2.800	0.5
96	146	1.900	0.5

### Center of mass correction removed



### Pu(240)

Functional	Binding Energy	First Barrier Height
SLy4	1801.5	11.9
SkM*	1804.3	9.4
UNEDF0	1811.8	9.6
UNEDF1	1811.8	6.8
Exp	1813.5	6.1

See the talk of N. Schunck



# NEXT UNEDF FUNCTIONAL

## Single-Particle Energies

### Single-particle splitting candidates

Neutrons		Protons	
<sup>40</sup> Ca	f <sub>5/2</sub> - f <sub>7/2</sub>	<sup>40</sup> Ca	f <sub>7/2</sub> - d <sub>3/2</sub>
<sup>40</sup> Ca	f <sub>7/2</sub> - d <sub>3/2</sub>		
<sup>48</sup> Ca	f <sub>5/2</sub> - f <sub>7/2</sub>	<sup>48</sup> Ca	f <sub>5/2</sub> - f <sub>7/2</sub>
<sup>132</sup> Sn	h <sub>9/2</sub> - h <sub>11/2</sub>	<sup>132</sup> Sn	g <sub>7/2</sub> - g <sub>9/2</sub>
<sup>208</sup> Pb	i <sub>11/2</sub> - i <sub>13/2</sub>	<sup>208</sup> Pb	h <sub>9/2</sub> - h <sub>11/2</sub>

### Goal:

- Improving spectroscopic properties of the functional
- Resolving the ability to optimize spin-orbit and tensor terms simultaneously

Using odd-even mass differences after applying a blocking procedure

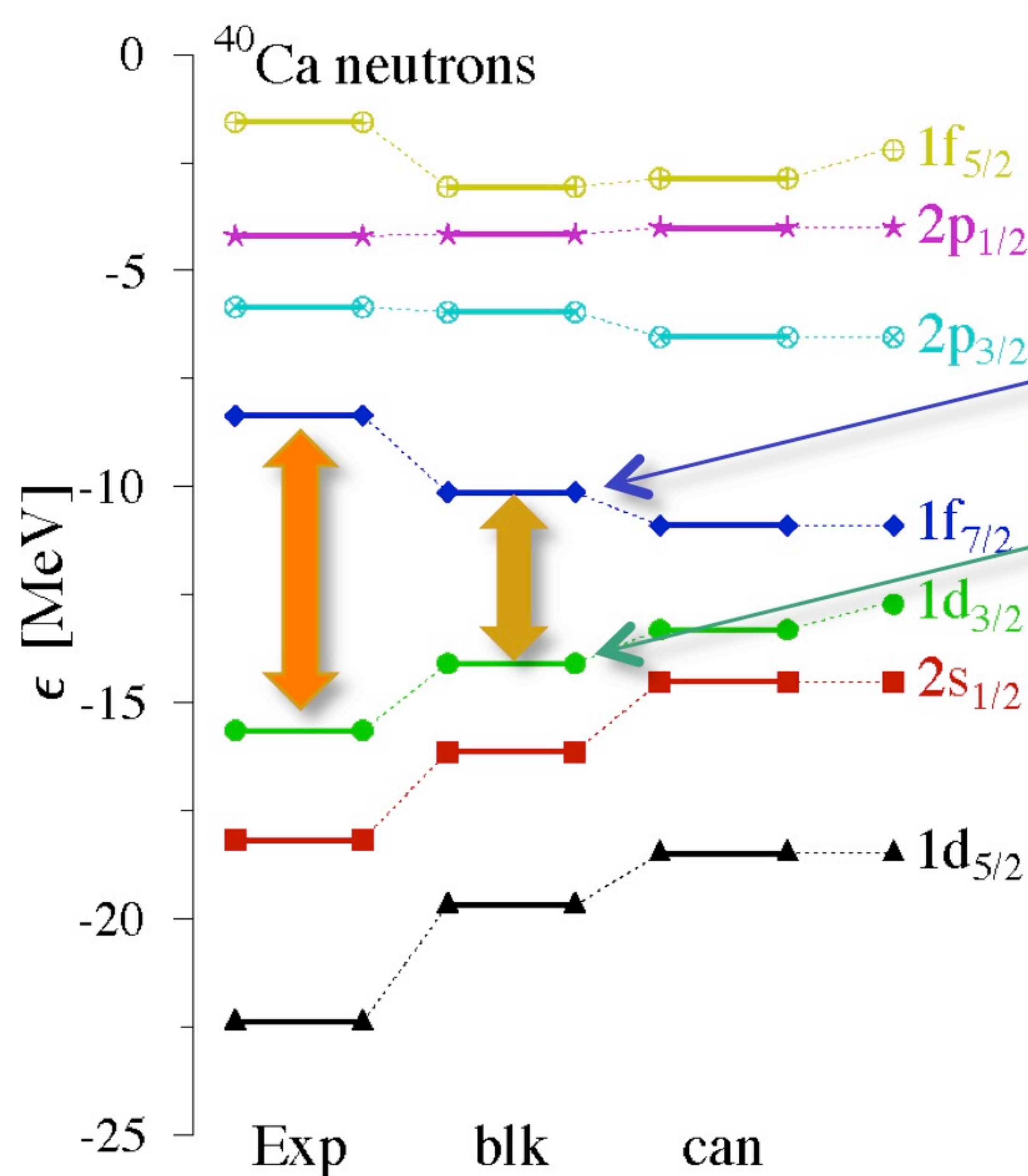
### Calculating:

$$e_{f7/2} = E_{\text{HF}}[\text{}^{40}\text{Ca}] - E_{\text{HF}}[\text{}^{41}\text{Ca} + \text{blocking: } f7/2]$$

$$e_{d3/2} = E_{\text{HF}}[\text{}^{40}\text{Ca}] - E_{\text{HF}}[\text{}^{39}\text{Ca} + \text{blocking: } d3/2]$$

### Fitting:

$$\Delta e = e_{f7/2} - e_{d3/2}$$

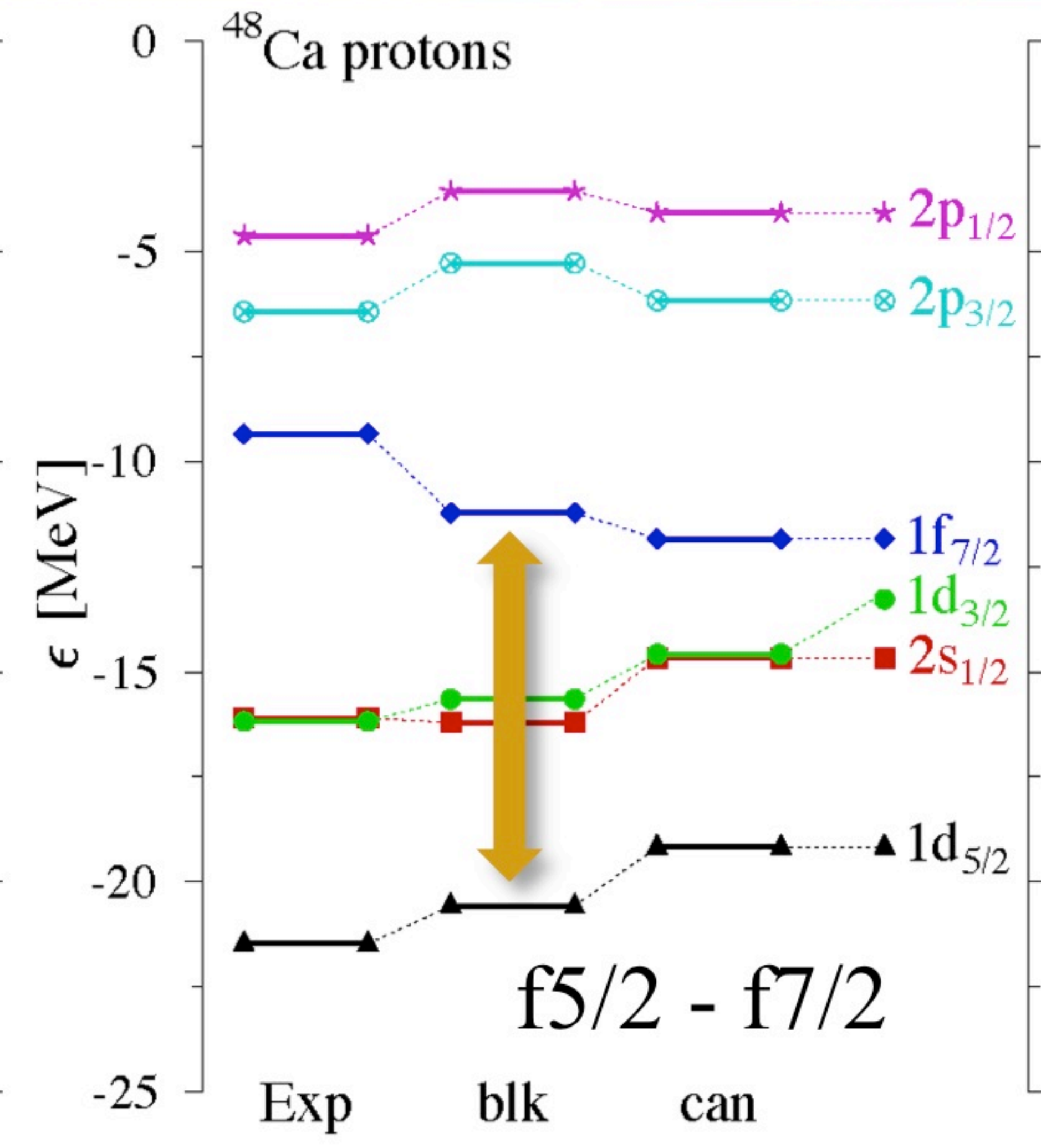
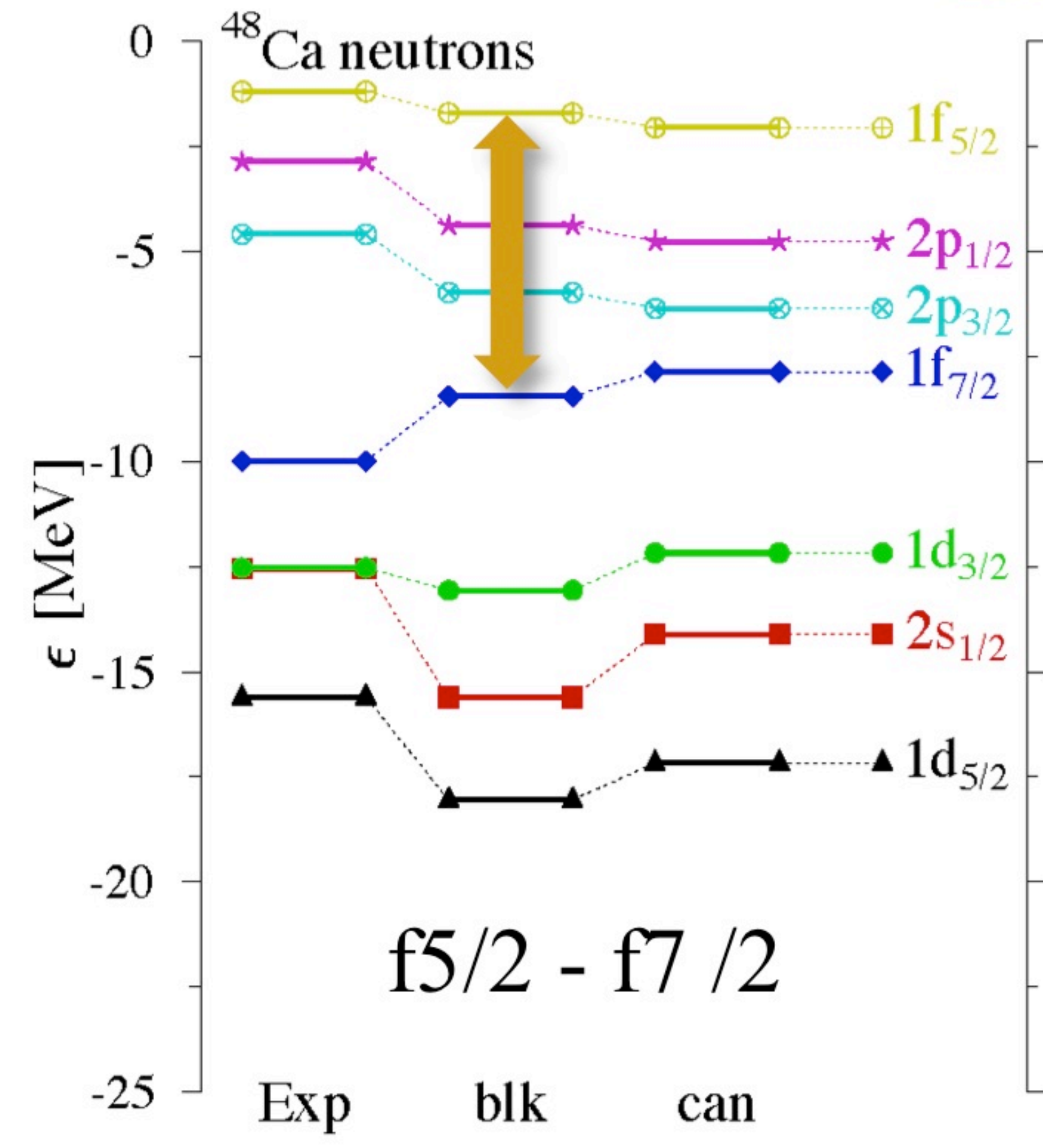
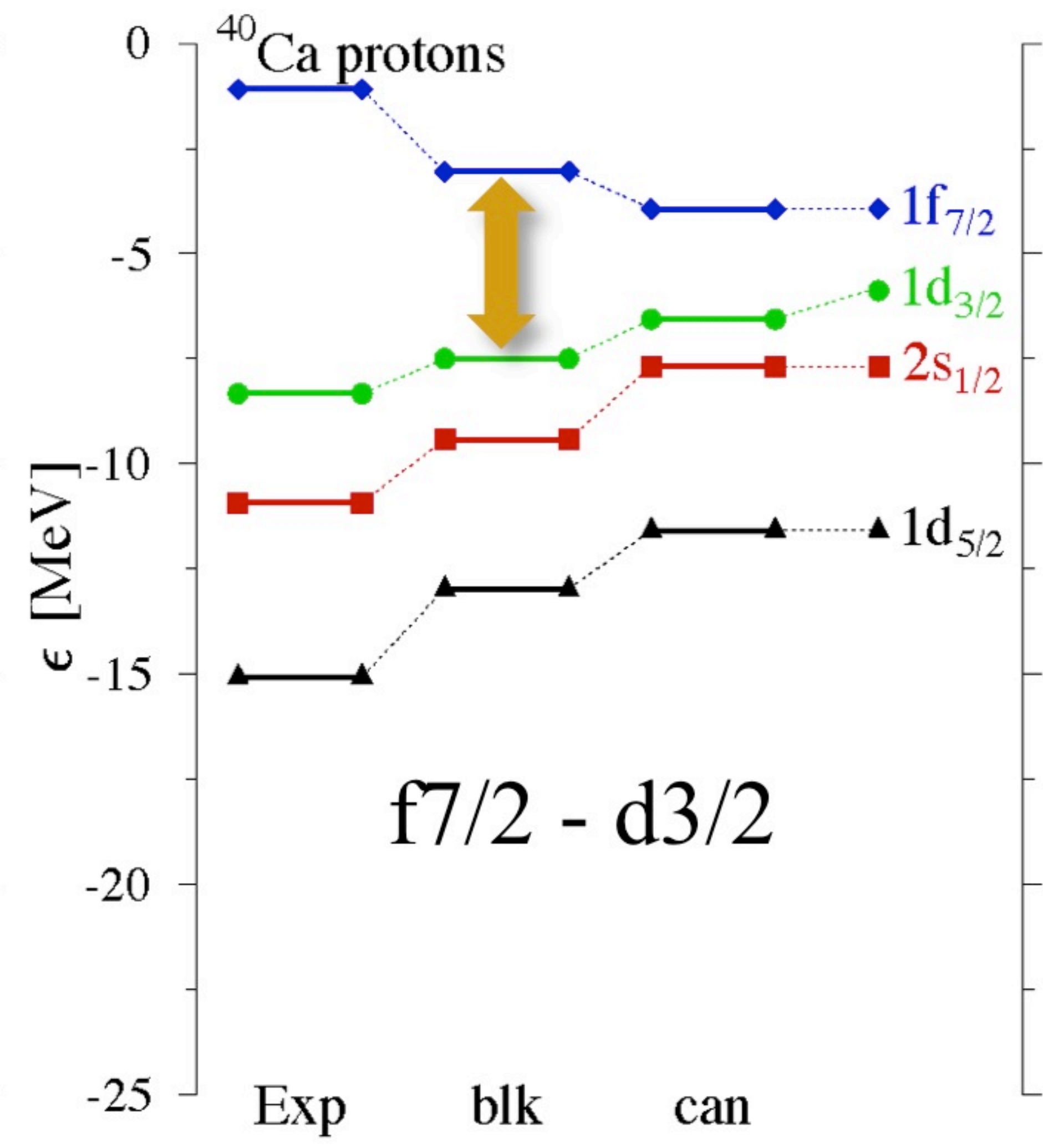
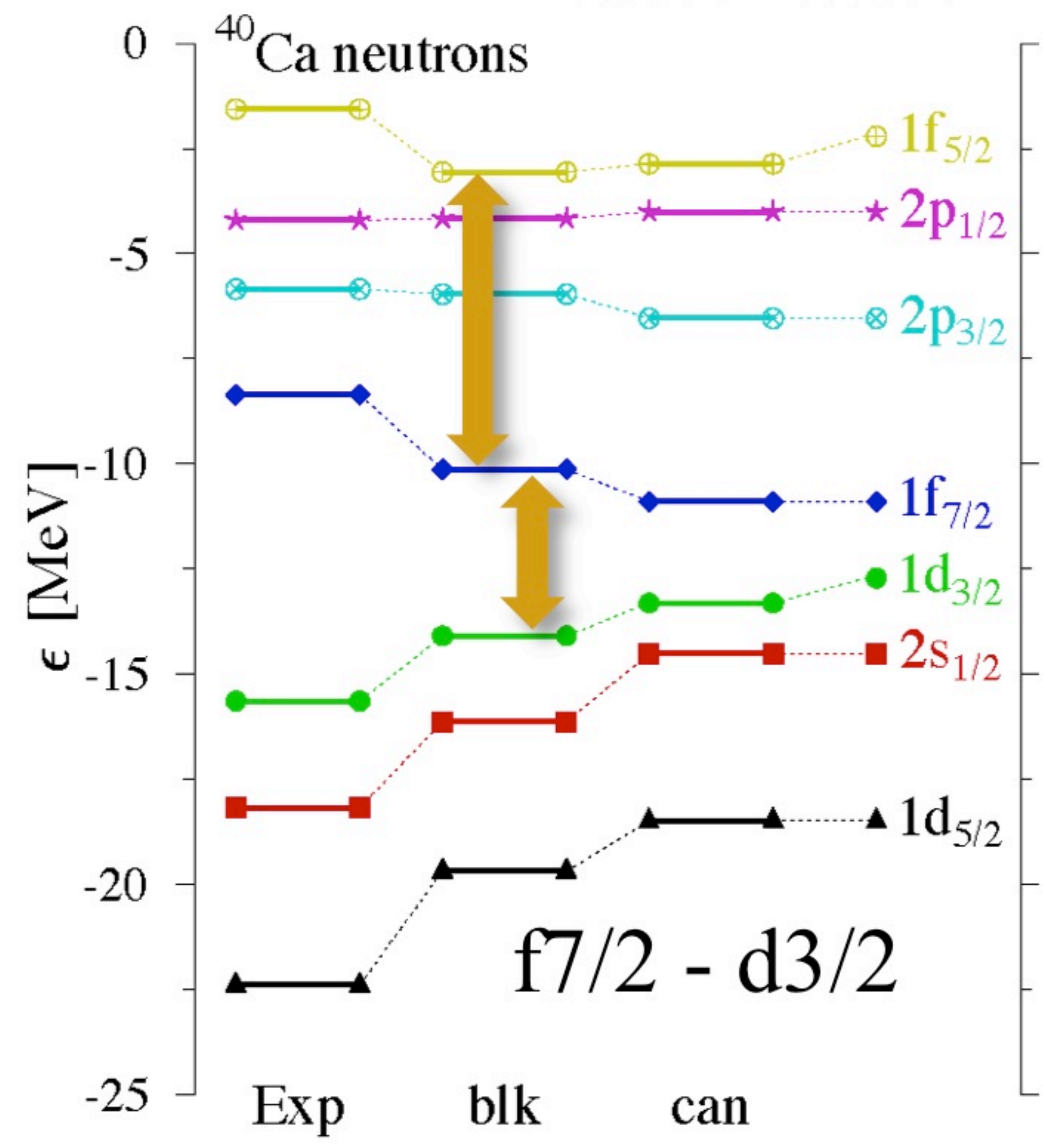




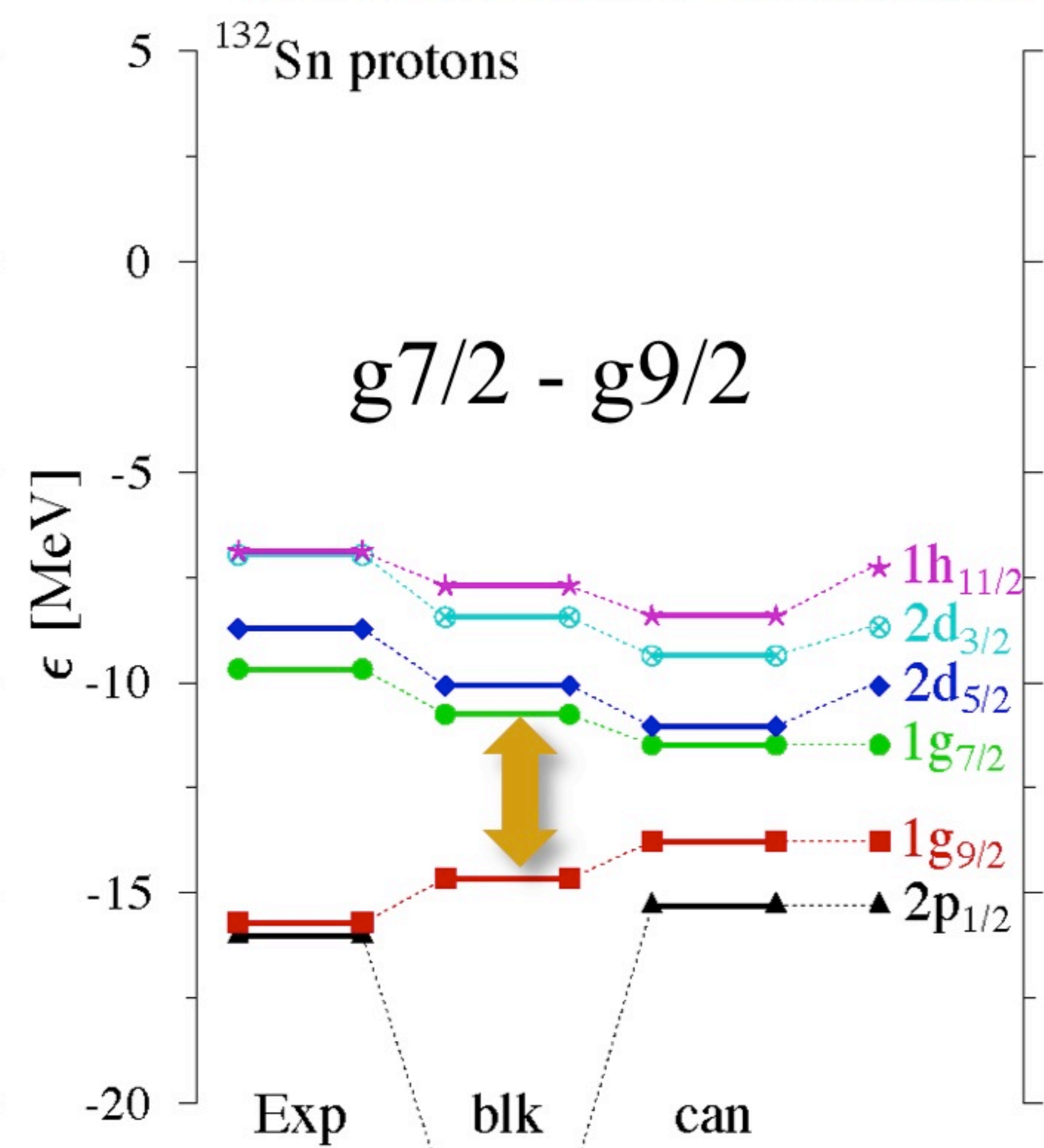
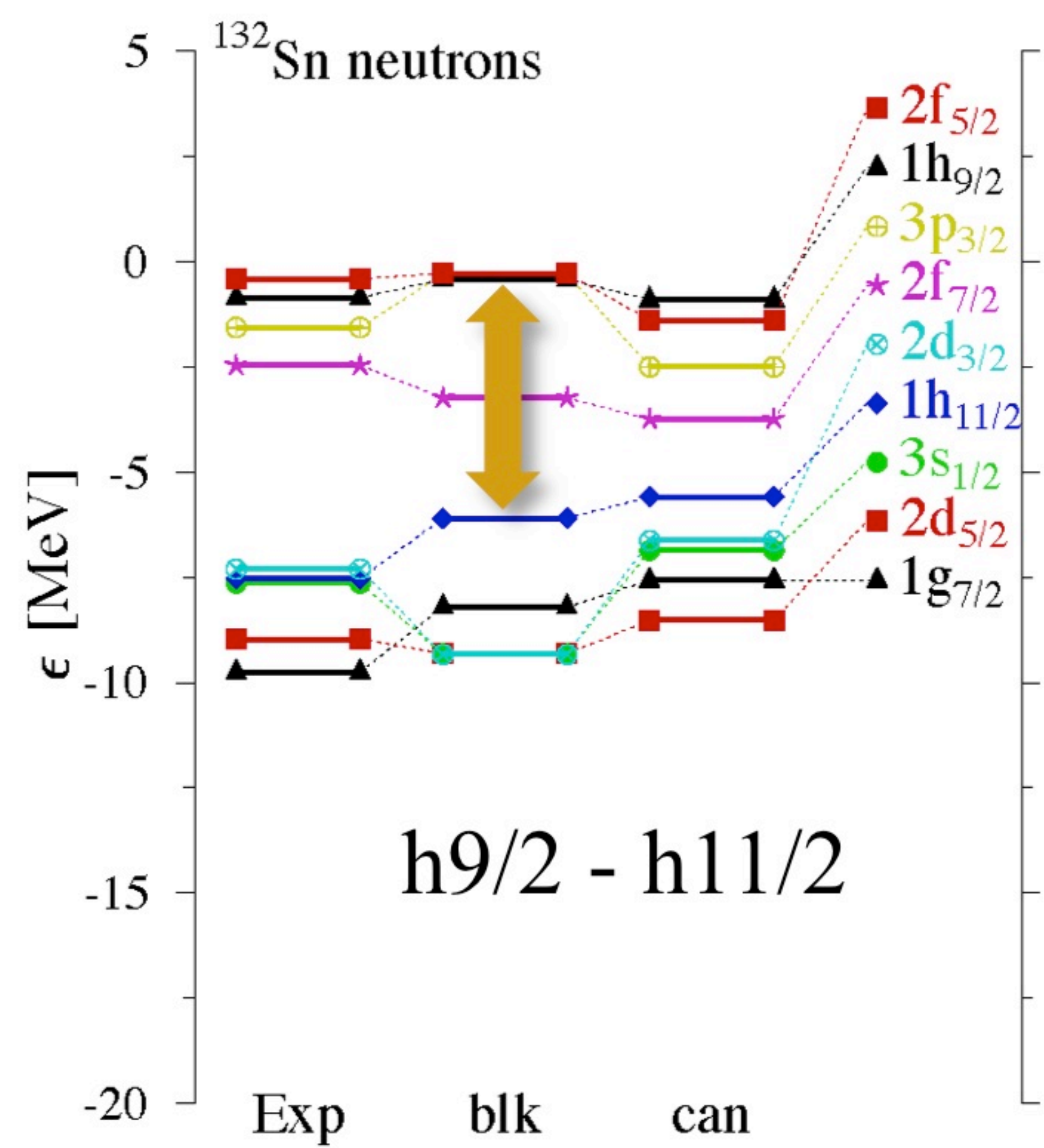
# NEXT UNEDF FUNCTIONALS

## Single-Particle Energies

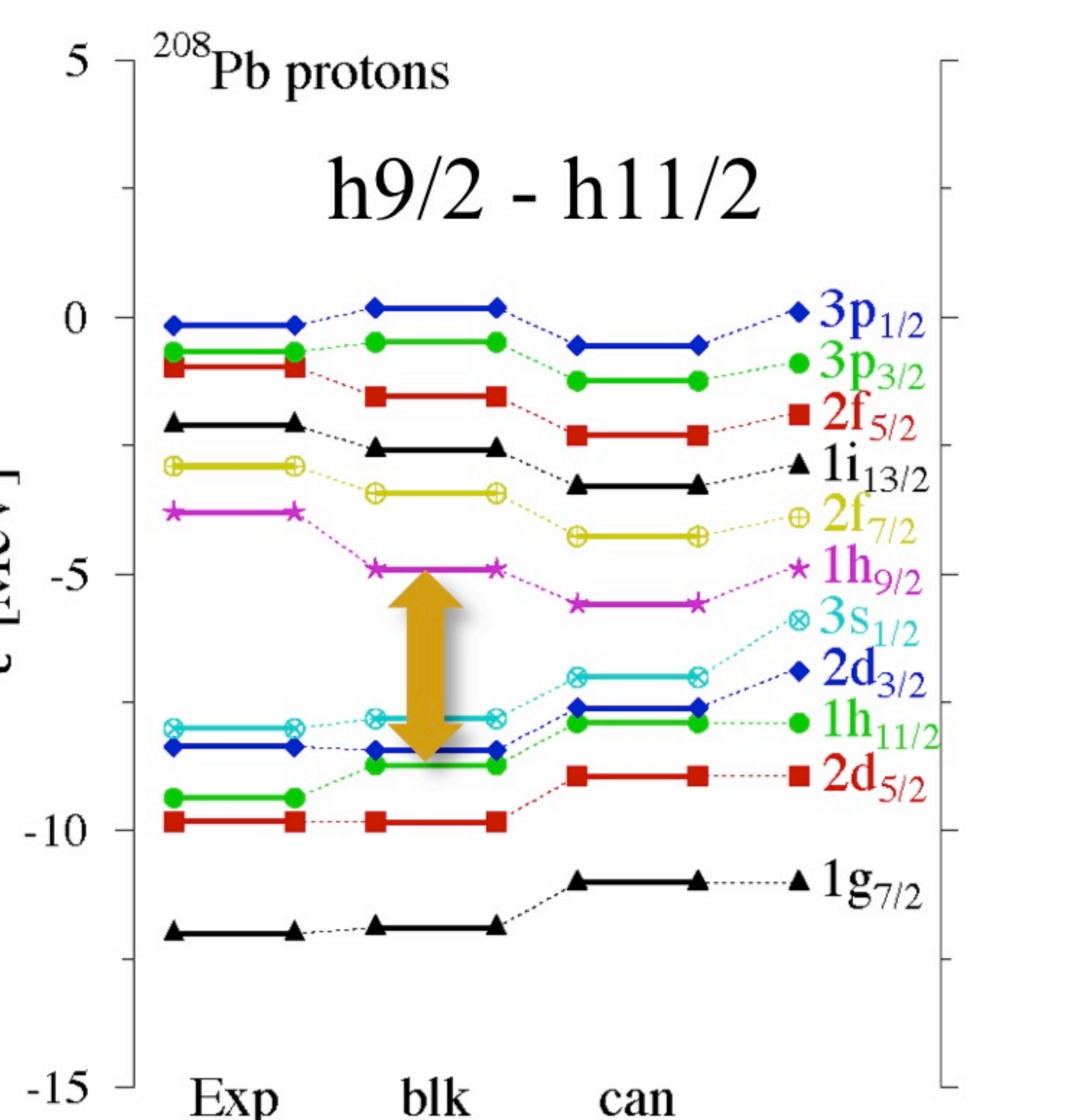
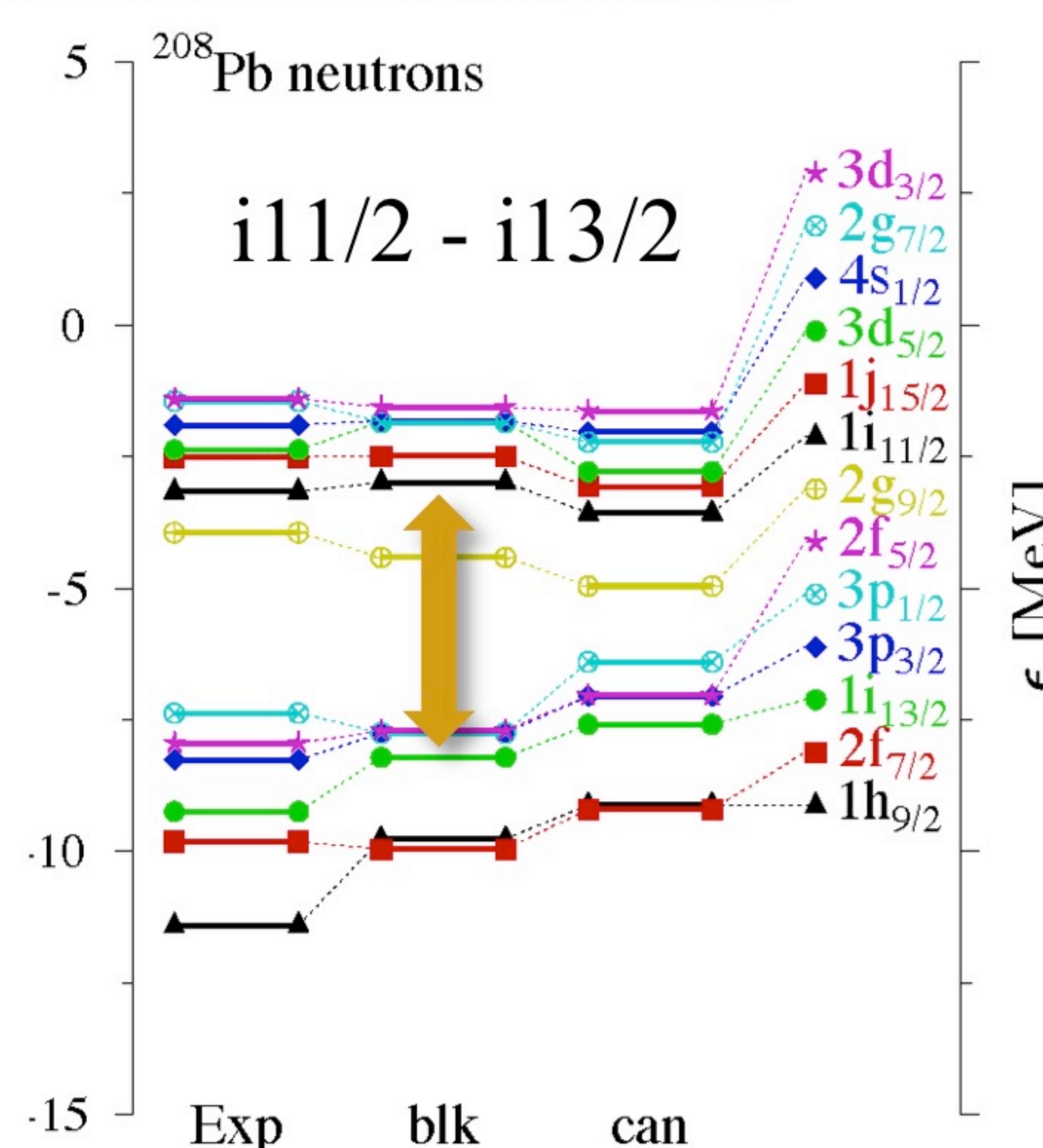
$f_{5/2} - f_{7/2}$



$h_{9/2} - h_{11/2}$



$i_{11/2} - i_{13/2}$



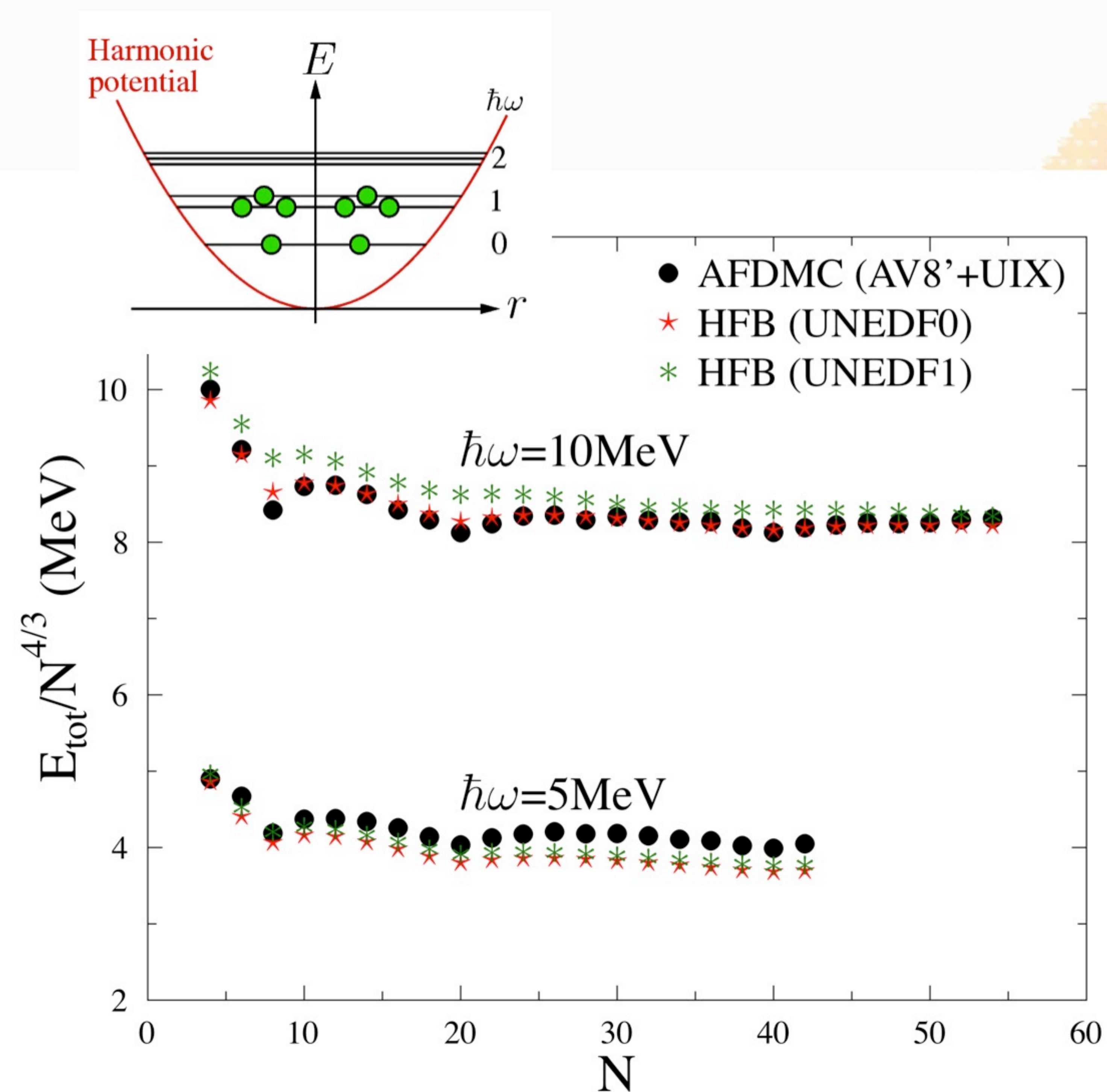


# NEXT UNEDF FUNCTIONAL

## Neutron Drops and Giant Resonances

### Including pseudo data form neutron drops in a trap

- Small strength of the HO trap (or Woods-Saxon trap ?)
- Large number N of the neutrons (CC-results ?)
- Total energies
- RMS radii
- Neutron infinite nuclear matter at low densities



See the talk of M. Kortelainen

### Including experimental data from giant resonances

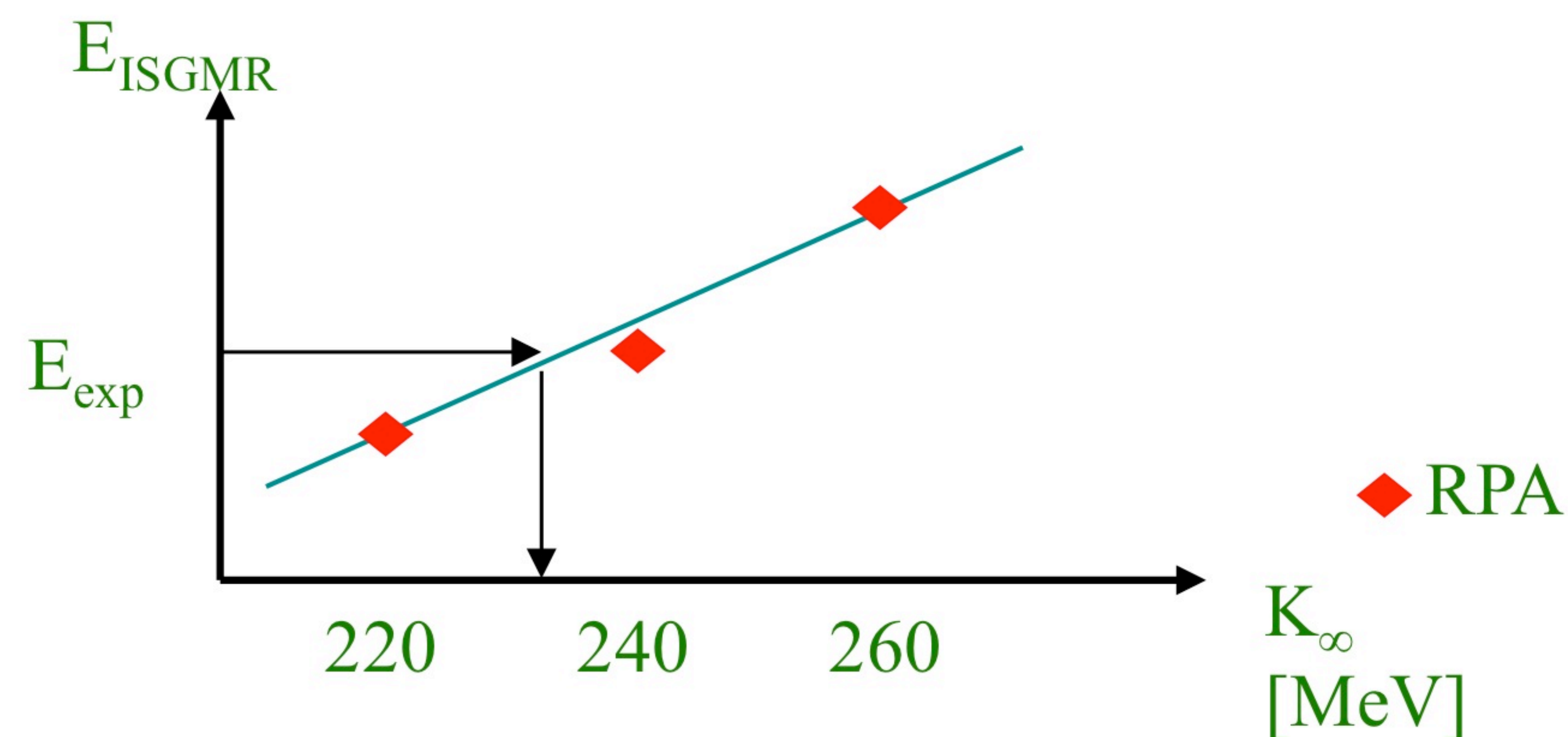
RPA SUM RULE APPROACH

Work in progress: E. Olsen

$$E_{ISGMR} = \sqrt{\frac{m(1)}{m(-1)}}$$

$$H' = H + \lambda r^2,$$

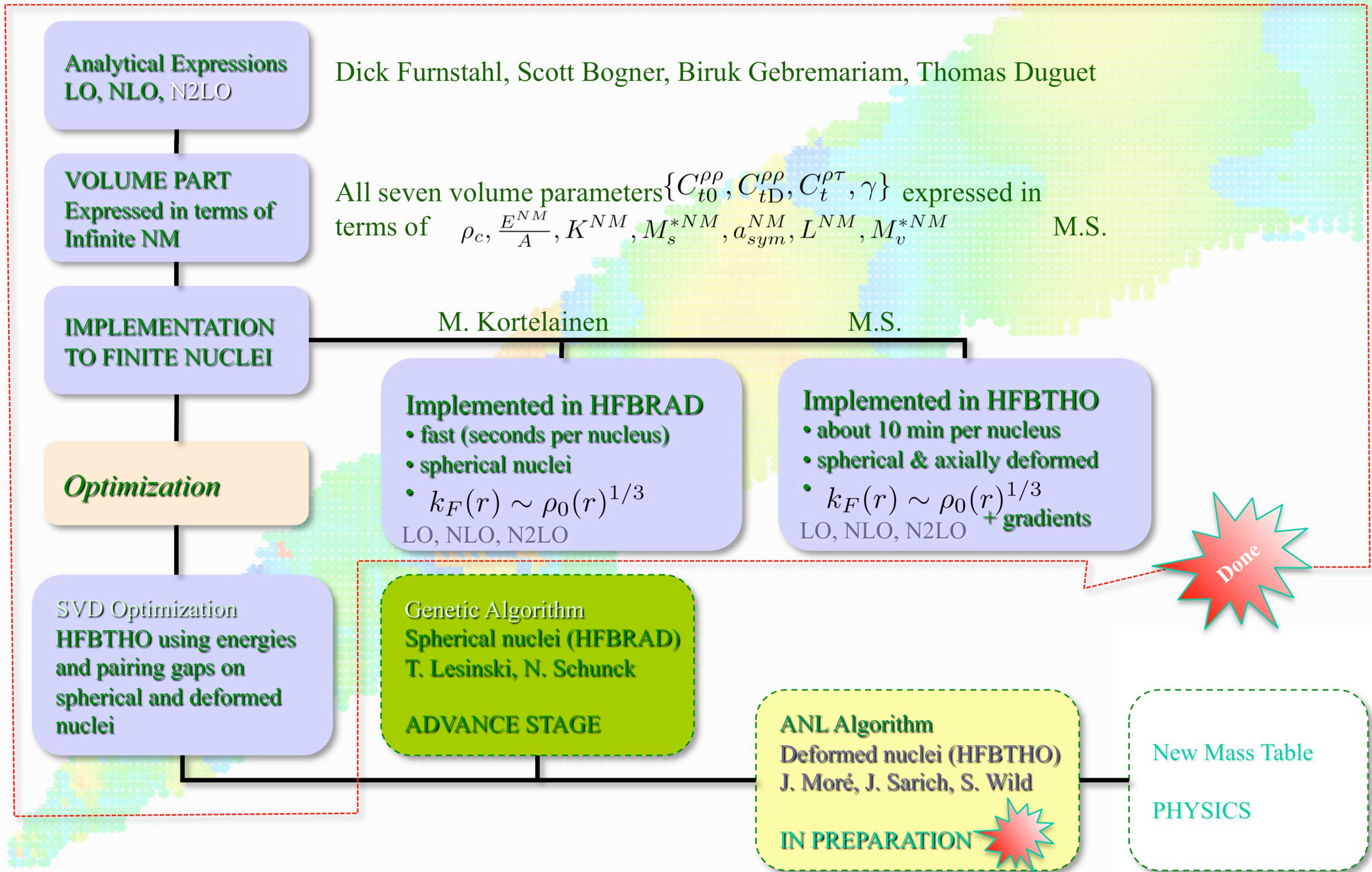
$$m(-1) = -\frac{1}{2} \frac{\delta \langle r^2 \rangle}{\delta \lambda} = \frac{1}{2} \frac{\delta^2 \langle H \rangle}{\delta \lambda^2}$$







# NEXT UNEDF FUNCTIONAL DME Functional Optimization







# HFBTHO ENHANCEMENT

- ✓ Reflection symmetry restriction removed – remains optional
- ✓ Blocking individual quasi-particle state implemented
- ✓ General functionals implemented - including DME functional
- ✓ Handle neutron drops in a trap – optional
- ✓ Separable pairing interaction implemented - optional
- ✓ FAM QRPA implemented – optional
- ✓ Improved performance:

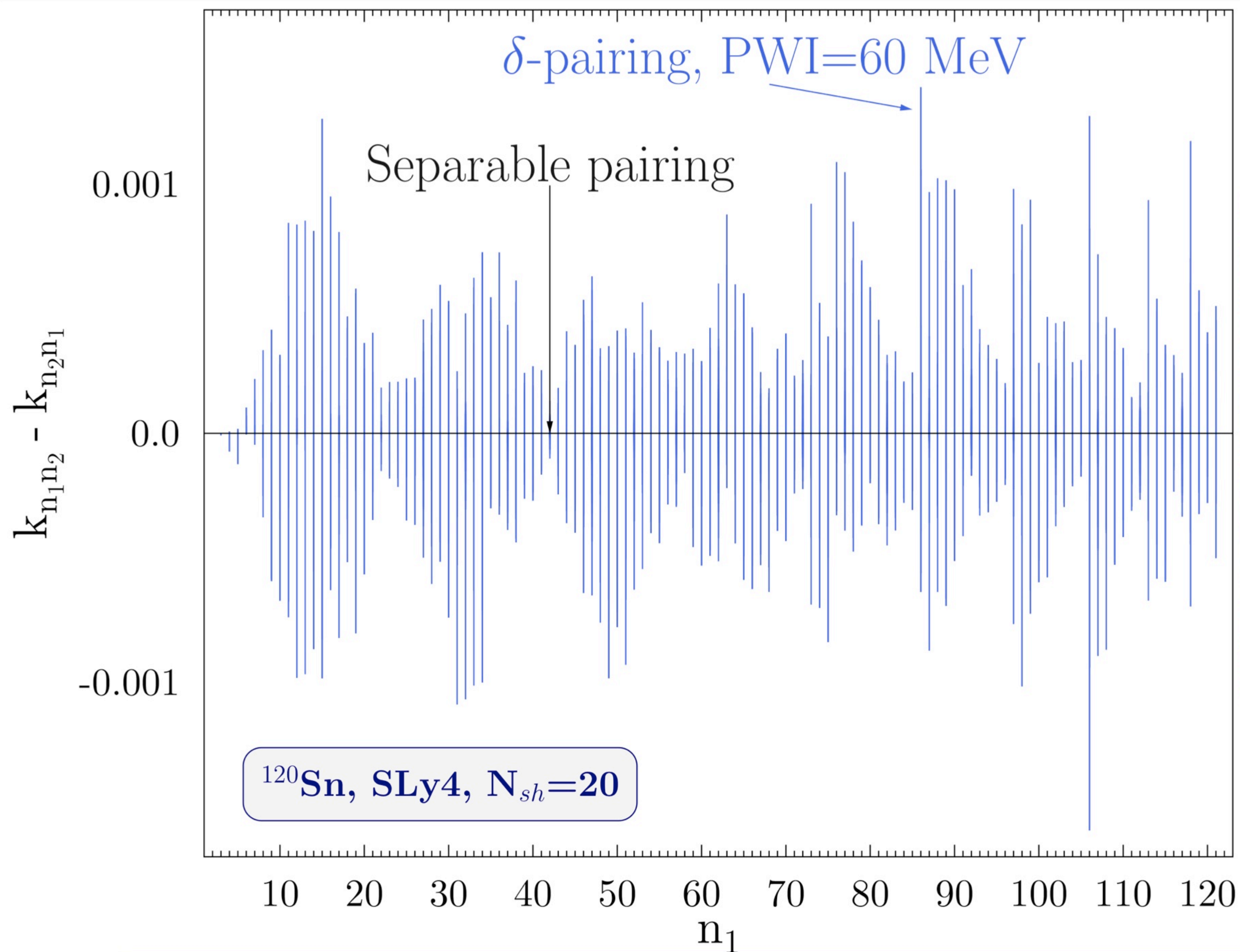
**Single core:** - 3.5 sec CPU time per iteration at  $N_{sh}=20$   
- about 20-30 iterations to converge

**Open MP:** Jason Sarich is working to reinforce the OpenMP implementation he did for the previous version (additional acceleration of 5-6 times expected)

See the talk of J. Sarich



# SEPARABLE PAIRING



The computational overhead is about 1 sec CPU time per iteration mainly due to the full HFB space used

$$\langle k | V_{sep} | k' \rangle = -G p(k) p(k')$$

$$p(k) = e^{-a^2 k^2}$$

*Y. Tian, Z. Ma, P. Ring, Phys.Rev. C 80, 024313 (2009)*

Avoids the non-unitarity of the Bogoliubov transformation due to the quasi-particle space truncation

J. Dobaczewski, P. Borycki, W. Nazarewicz, M. Stoitsov, EPJ A25 (2005) 541





# FAM QRPA

For years there is intensive effort to develop efficient tool for performing QRPA calculations for deformed nuclei. Finite Amplitude Method (FAM) is such a tool suggested by T. Nakatsukasa

## SHORT HISTORY

### Demonstration of FAM efficiency in RPA calculations

T. Nakatsukasa, T. Inakura and K. Yabana, Phys. Rev. C 76 024318 (2007).

T. Inakura, T. Nakatsukasa and K. Yabana, Phys. Rev. C 80 044301 (2009).

### Demonstration of FAM efficiency in QRPA calculations for spherical nuclei

P. Avogadro, T. Nakatsukasa, Finite amplitude method for the quasi-particle random-phase approximation, arXiv:1104.3692 (2011)

## PRESENT DEMONSTRATION

M.S., M. Kortelainen, T. Nakatsukasa, W. Nazarewicz, C. Losa

- ✧ New efficient way of solving FAM equations based on Broyden method
- ✧ Complete implementation within HFBTHO solver
- ✧ Benchmark comparison with standard QRPA results with pairing and deformation



**QRPA:** 
$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ -Y \end{pmatrix}$$

**FAM QRPA:** 
$$(E_\mu + E_\nu) X_{\nu\mu} + \delta H_{\nu\mu}^{20}(\omega) = \omega X_{\nu\mu},$$

$$(E_\mu + E_\nu) Y_{\nu\mu}^* + \overline{\delta H}_{\nu\mu}^{20}(\omega) = -\omega Y_{\nu\mu}^*,$$



$$\delta H^{20}(\omega) = U^\dagger \delta h(\omega) V^* - V^\dagger \overline{\delta h}^*(\omega) U^* - V^\dagger \overline{\delta \Delta}^*(\omega) V^* + U^\dagger \delta \Delta(\omega) U^*,$$

$$\overline{\delta H}^{20}(\omega) = U^\dagger \overline{\delta h}(\omega) V^* - V^\dagger \delta h^*(\omega) U^* - V^\dagger \Delta^*(\omega) V^* + U^\dagger \overline{\delta \Delta}(\omega) U^*,$$



$$\mathcal{H} = \begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix}$$



$$\rho_\eta = (V + \eta U^* X^*)^* (V + \eta U^* Y)^T,$$

$$k_\eta = -(U + \eta V^* Y) (V + \eta U^* X^*)^\dagger,$$

$$\bar{\rho}_\eta = (V + \eta U^* Y)^* (V + \eta U^* X^*)^T,$$

$$\bar{k}_\eta = -(U + \eta V^* X^*) (V + \eta U^* Y)^\dagger.$$



$$\delta h(\omega) = \frac{h[\rho_\eta, k_\eta] - h[\rho, k]}{\eta},$$

$$\overline{\delta h}(\omega) = \frac{h[\bar{\rho}_\eta, \bar{k}_\eta] - h[\rho, k]}{\eta},$$

$$\delta \Delta(\omega) = \frac{\Delta[\rho_\eta, k_\eta] - \Delta[\rho, k]}{\eta},$$

$$\overline{\delta \Delta}(\omega) = \frac{\Delta[\bar{\rho}_\eta, \bar{k}_\eta] - \Delta[\rho, k]}{\eta},$$

**HFB**



## FAM QRPA EQUATIONS

$$(E_\mu + E_\nu - \omega) X_{\nu\mu} = -\delta H_{\nu\mu}^{20}(\omega) - F_{\nu\mu},$$

$$(E_\mu + E_\nu + \omega) Y_{\nu\mu} = -\overline{\delta H}_{\nu\mu}^{20*}(\omega) - F_{\nu\mu},$$



$$X_{\mu\nu} = -\frac{\delta H_{\mu\nu}^{20}(\omega) - F_{\mu\nu}^{20}}{E_\mu + E_\nu - \omega},$$

$$Y_{\mu\nu} = -\frac{\overline{\delta H}_{\mu\nu}^{20*}(\omega) - F_{\mu\nu}^{20*}}{E_\mu + E_\nu + \omega}.$$

## SELF-CONSISTENT ITERATIONS (Broyden mixing as in HFB)

$$X_{\mu\nu}^{(0)} = Y_{\mu\nu}^{(0)} = 0 \implies \delta H_{\mu\nu}^{20(0)}(\omega), \overline{\delta H}_{\mu\nu}^{20(0)}(\omega) \implies X_{\mu\nu}^{(1)}, Y_{\mu\nu}^{(1)} \implies \delta H_{\mu\nu}^{20(1)}(\omega), \overline{\delta H}_{\mu\nu}^{20(1)}(\omega), \dots$$

## COMPLEX FREQUENCY

$$\omega \rightarrow \omega + i\gamma$$



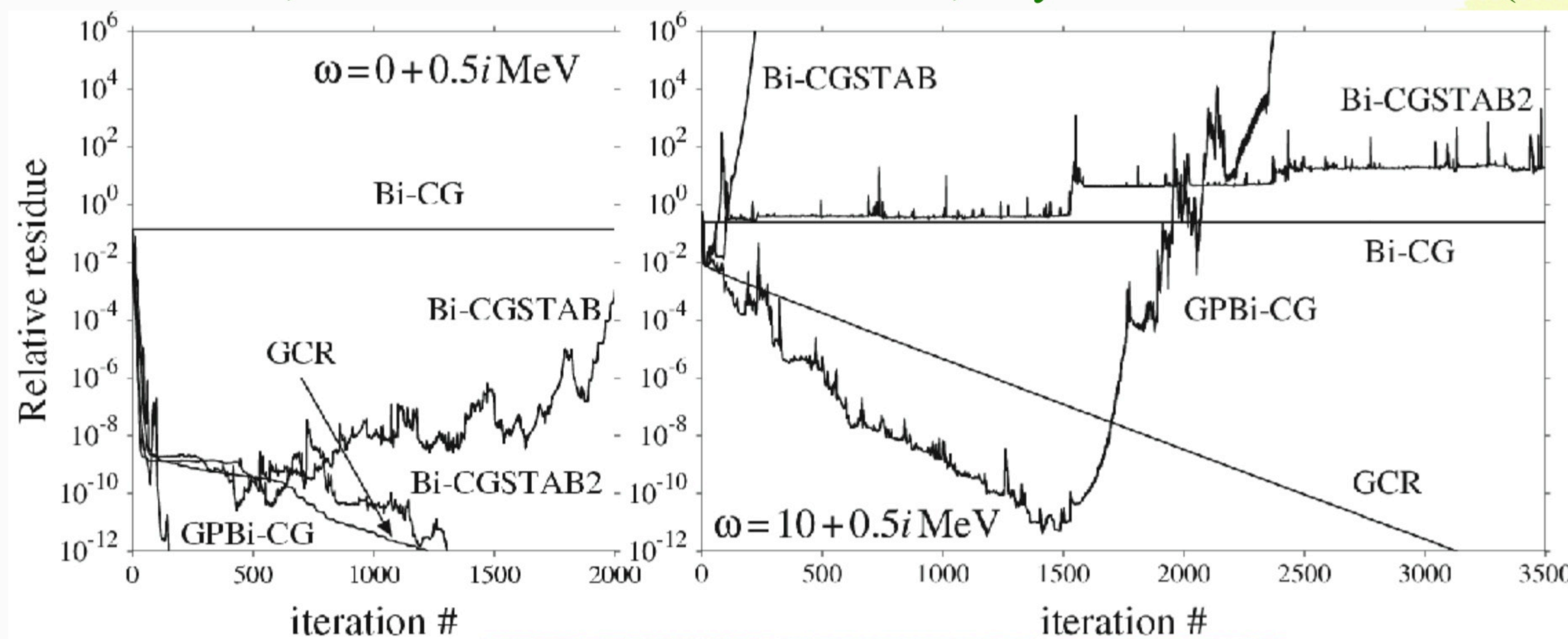
Lorenz smoothing

$$\Gamma = 2\gamma$$

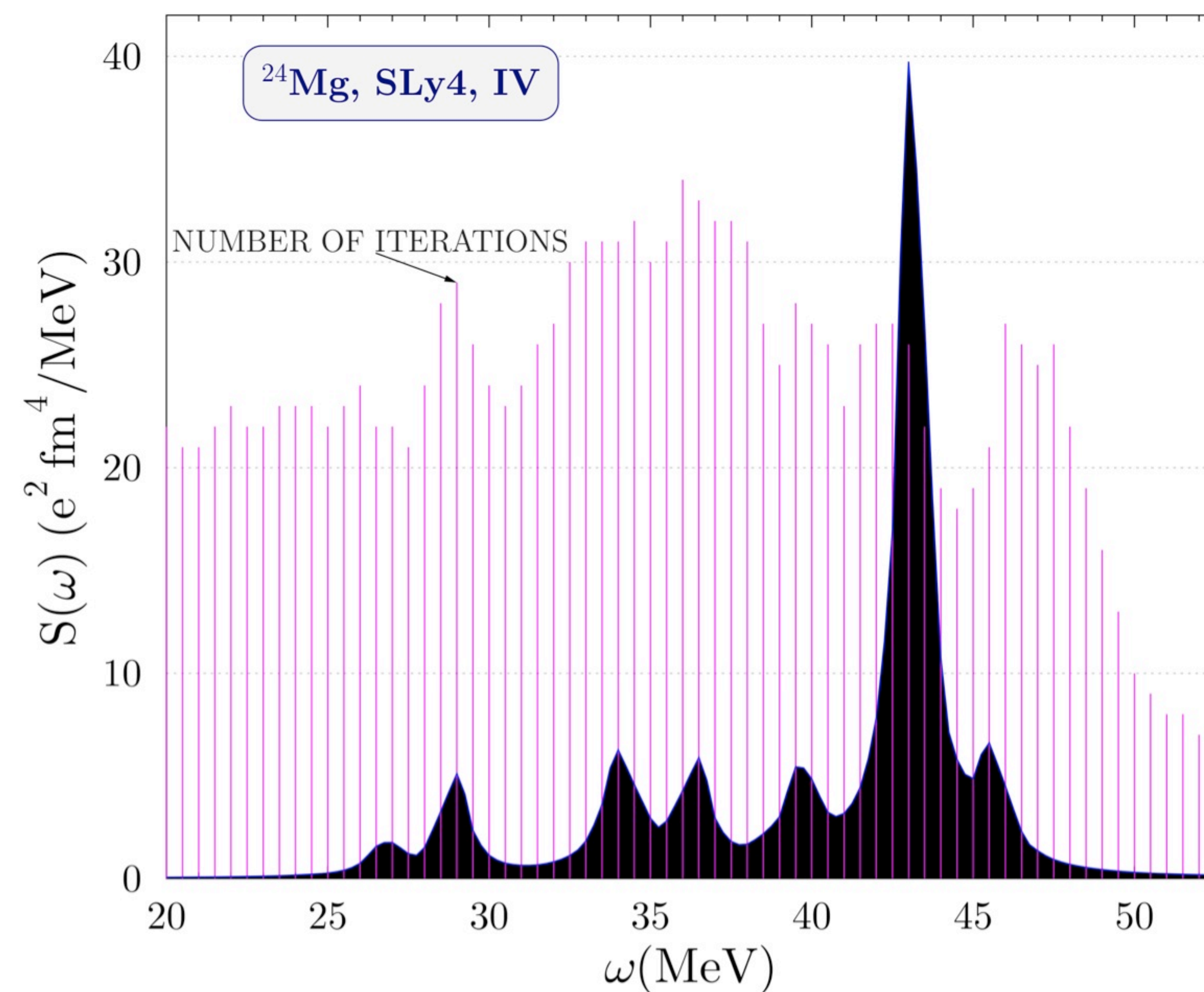
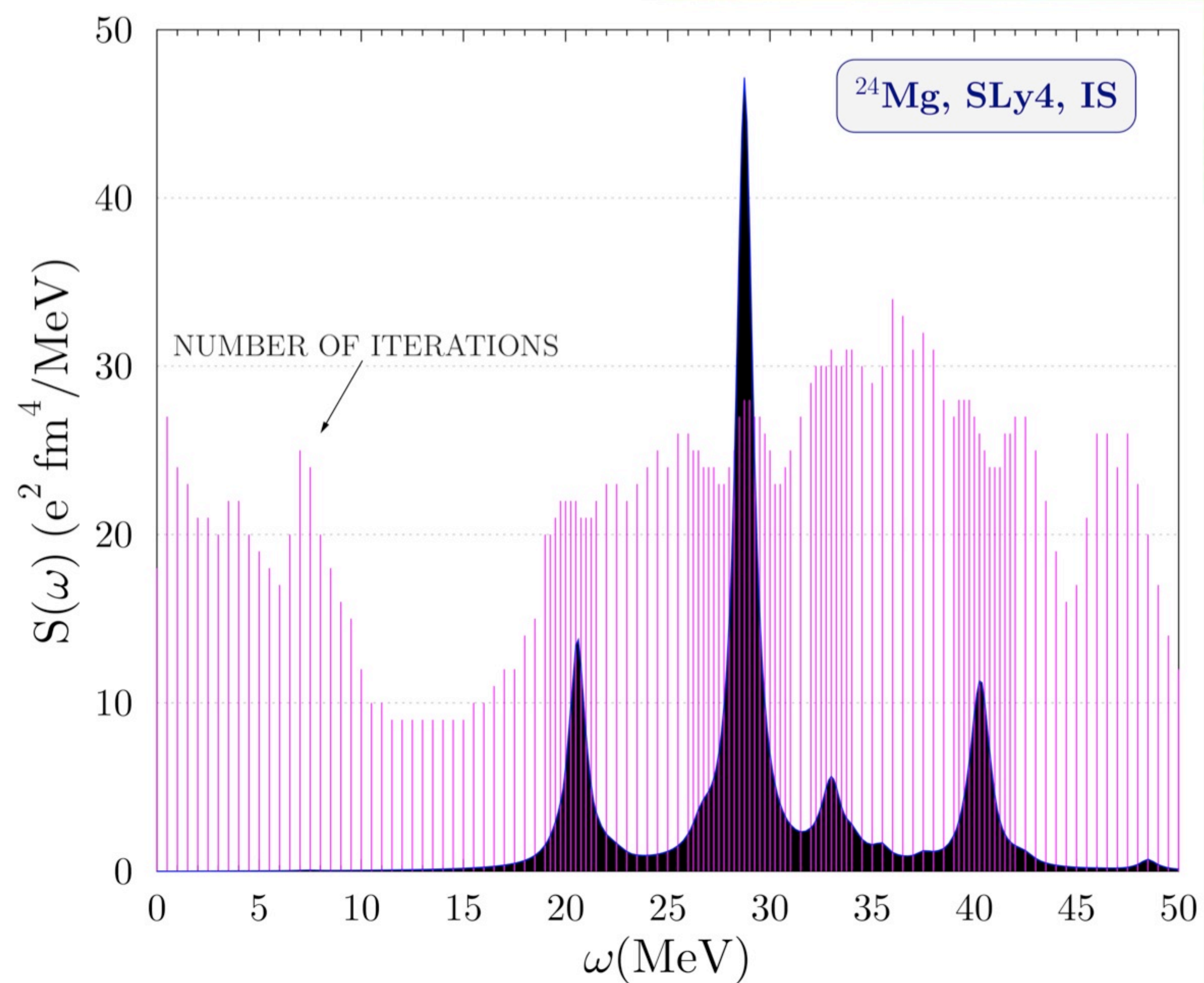


# FAM QRPA - NUMBER OF ITERATIONS

T. Inakura, T. Nakatsukasa and K. Yabana, Phys. Rev. C 80 044301 (2009).



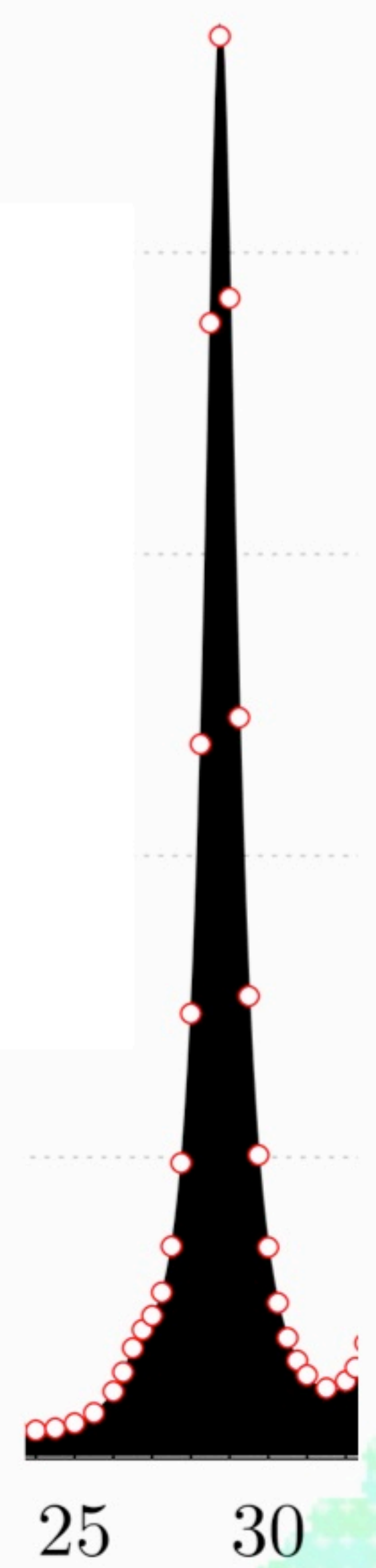
## PRESENT METHOD



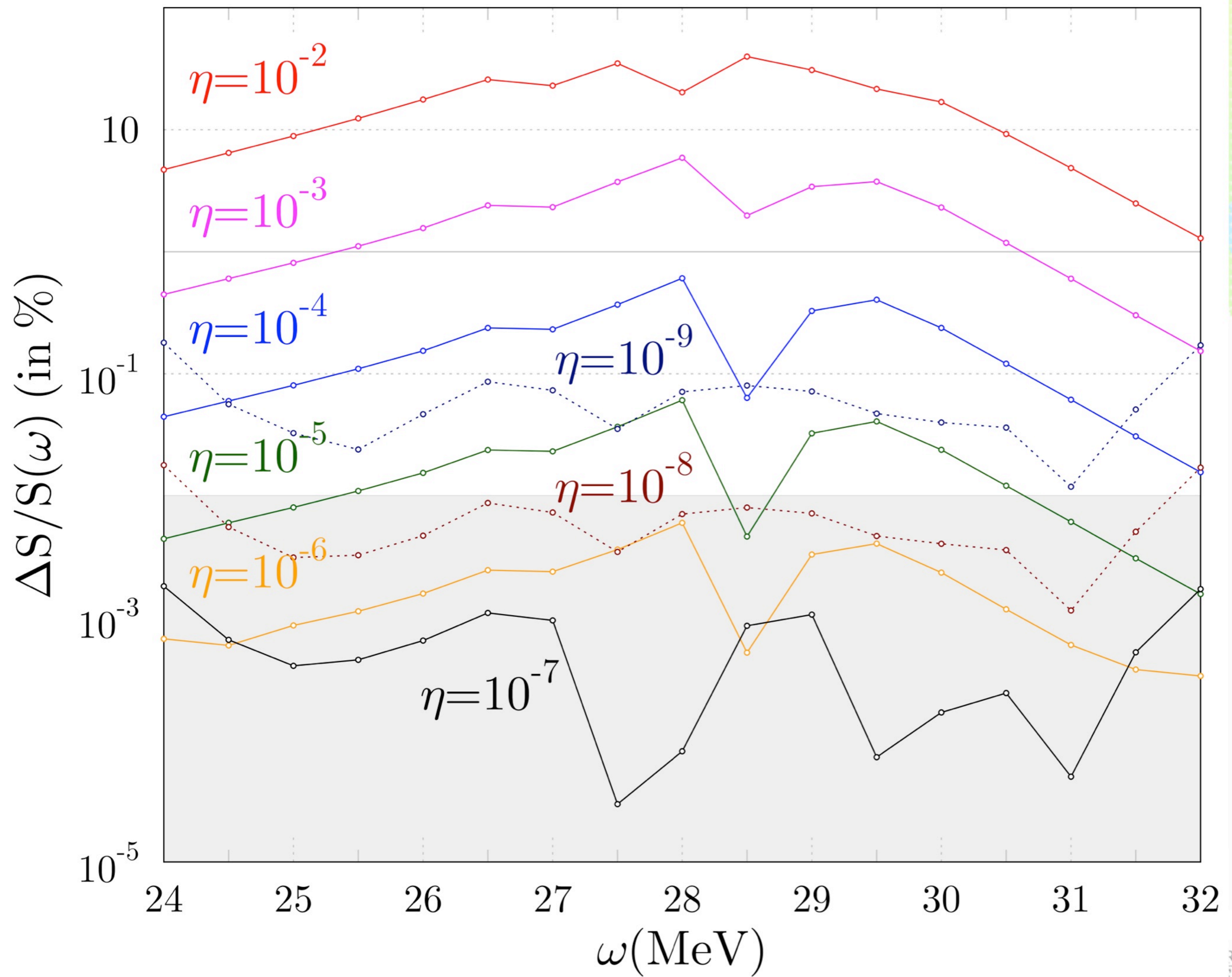


# FAM QRPA - ACCURACY

53



5







# QRPA – FAM (COMPARISON)

## QRPA:

Calculating and diagonalizing A,B matrices in the canonical basis following from the HFB solution obtained by the HFBTHO solver

C. Losa, A. Pastore, T. Dossing, E. Vigezzi, R.A. Broglia, *Phys. Rev. C* 81, 064307 (2010).

## FAM:

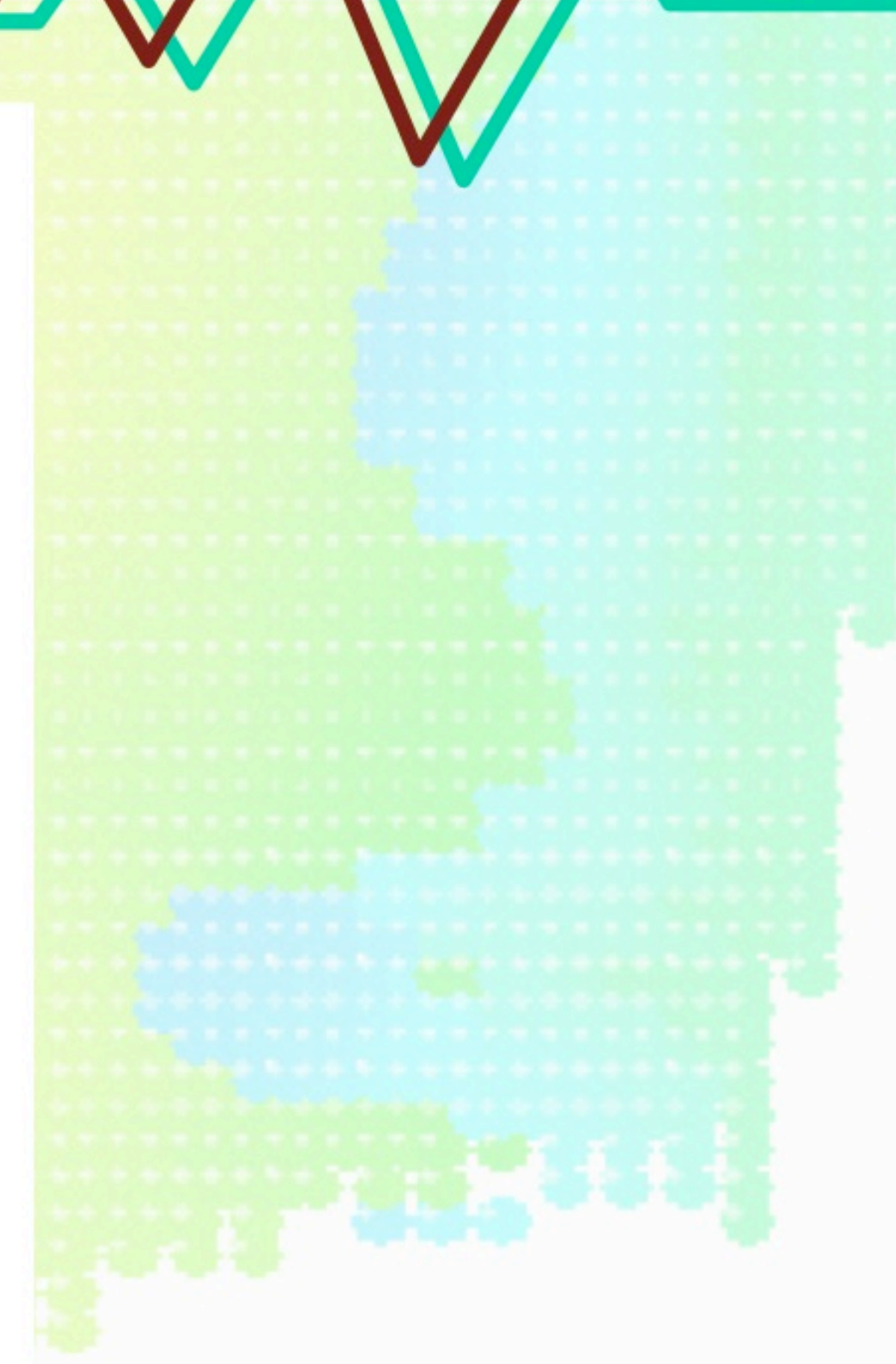
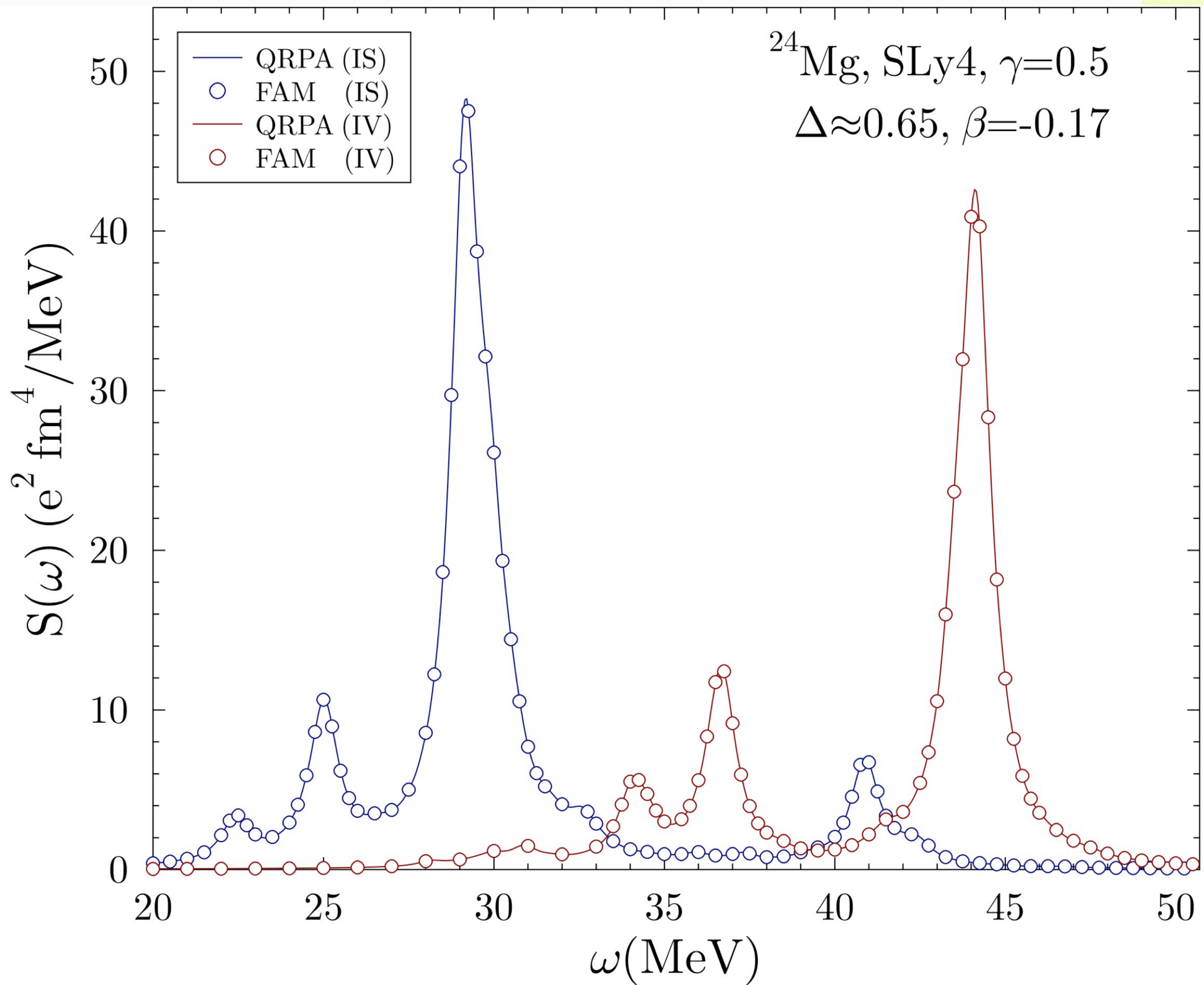
FAM QRPA module build on the top of HFBTHO solver

**SLY4, VOLUME PAIRING, ISGMR, IVGMR**



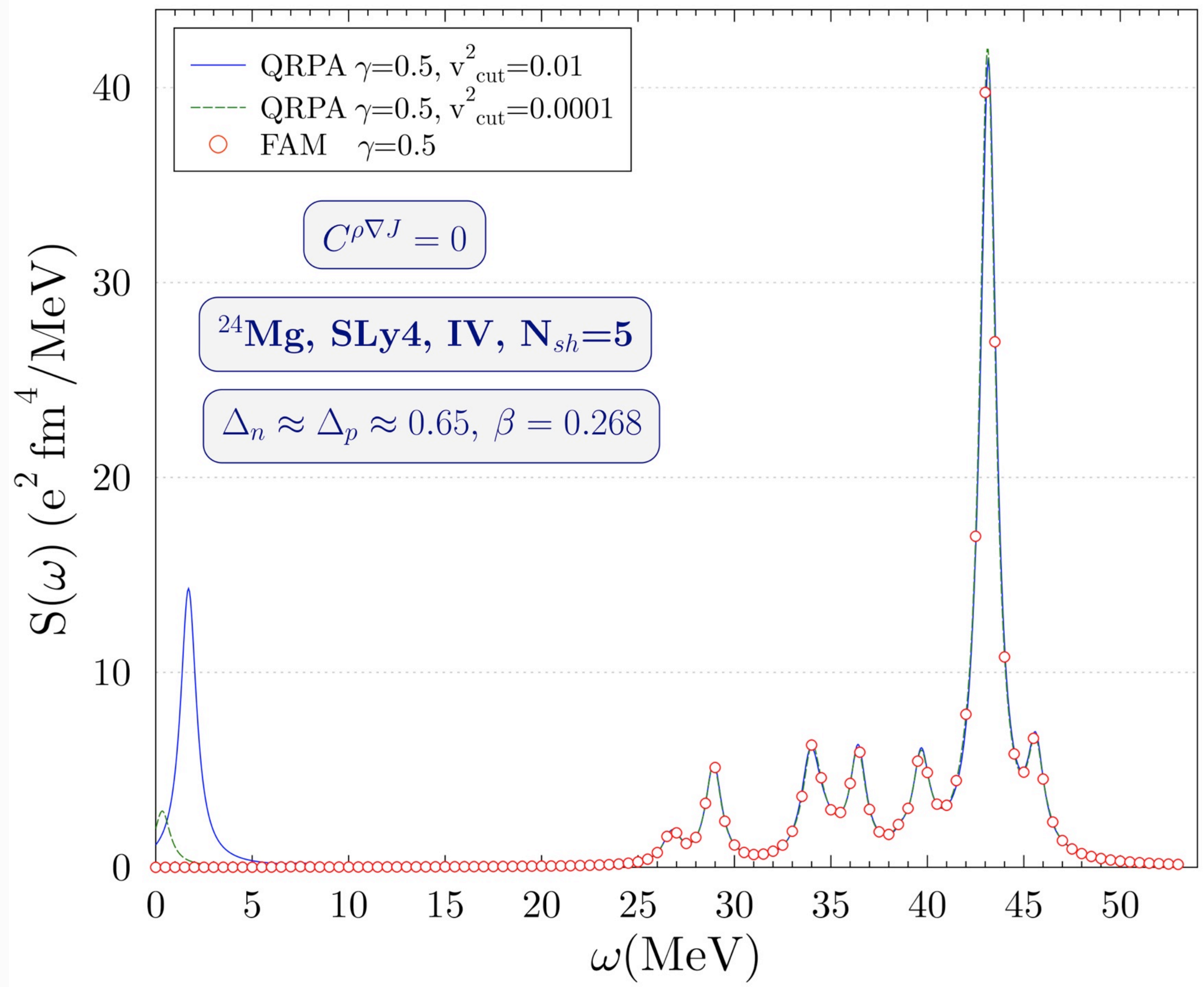
# QRPA versus FAM

## NUMERICALLY IDENTICAL RESULTS

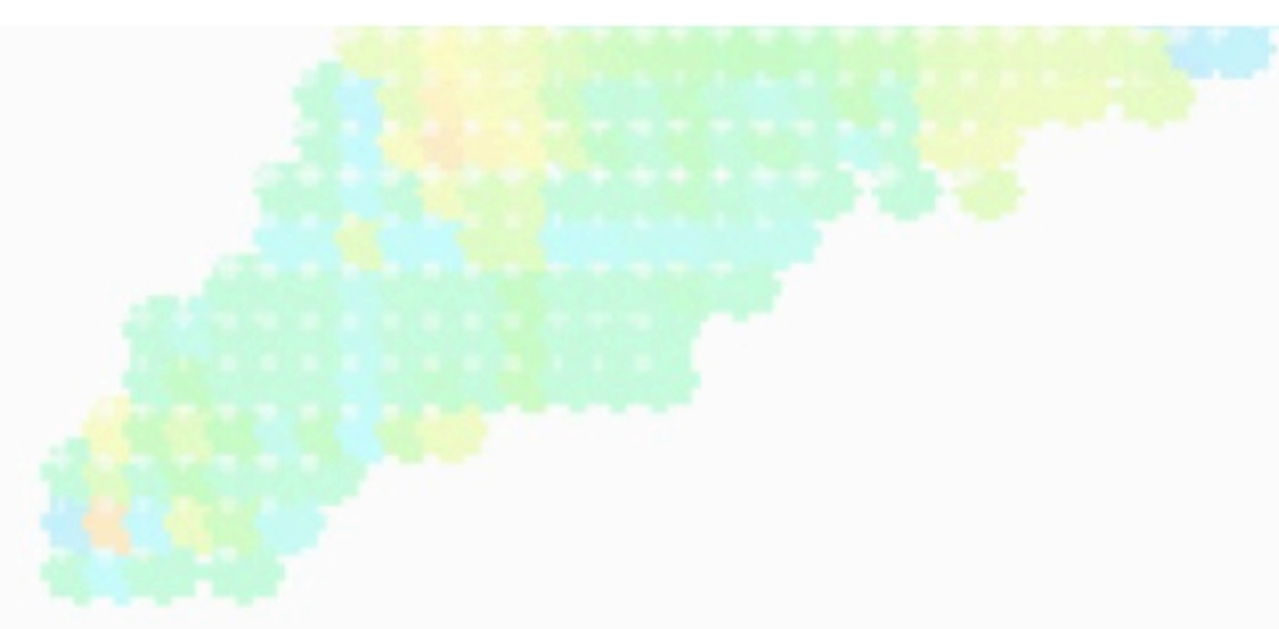




# QRPA TRUNCATIONS

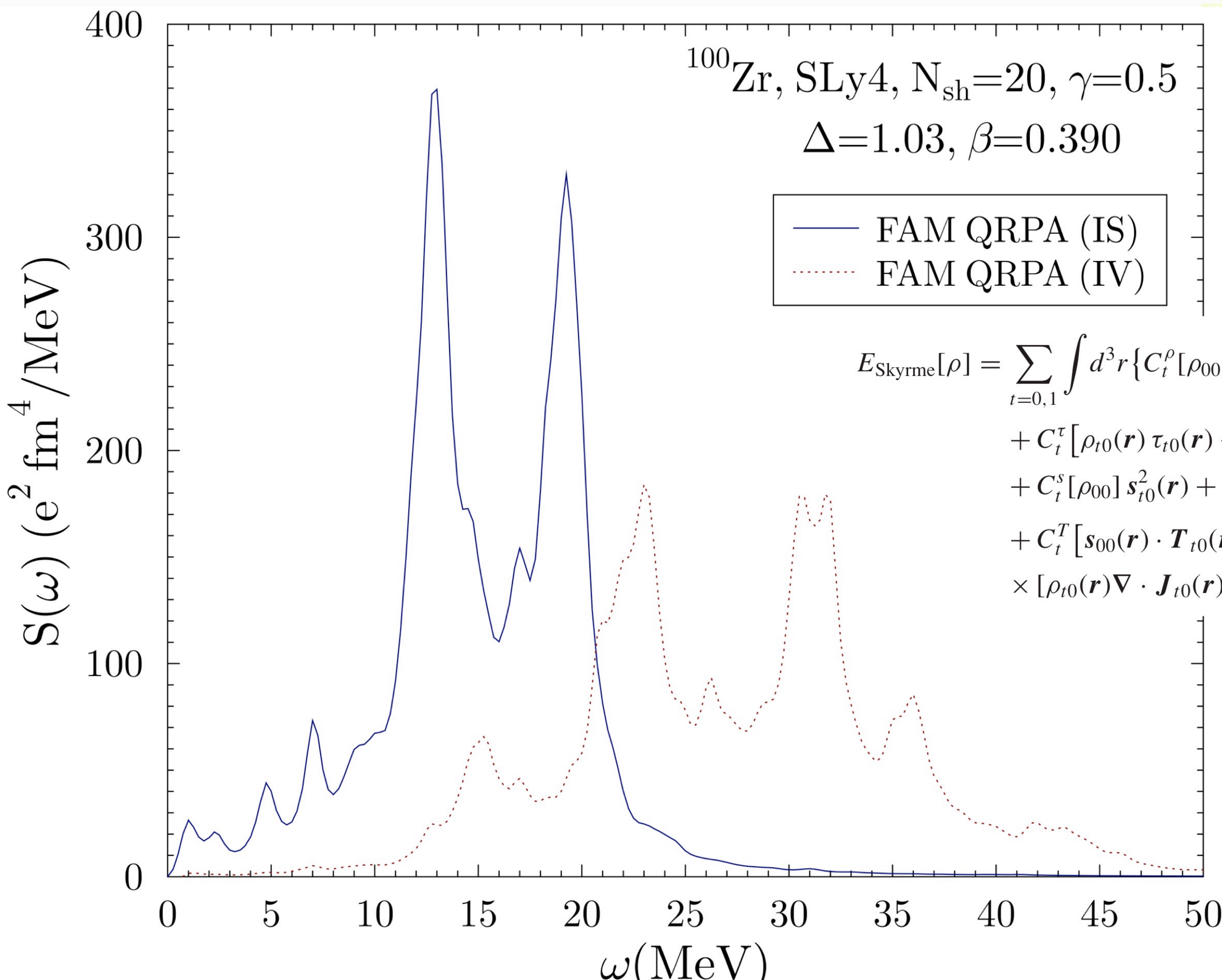
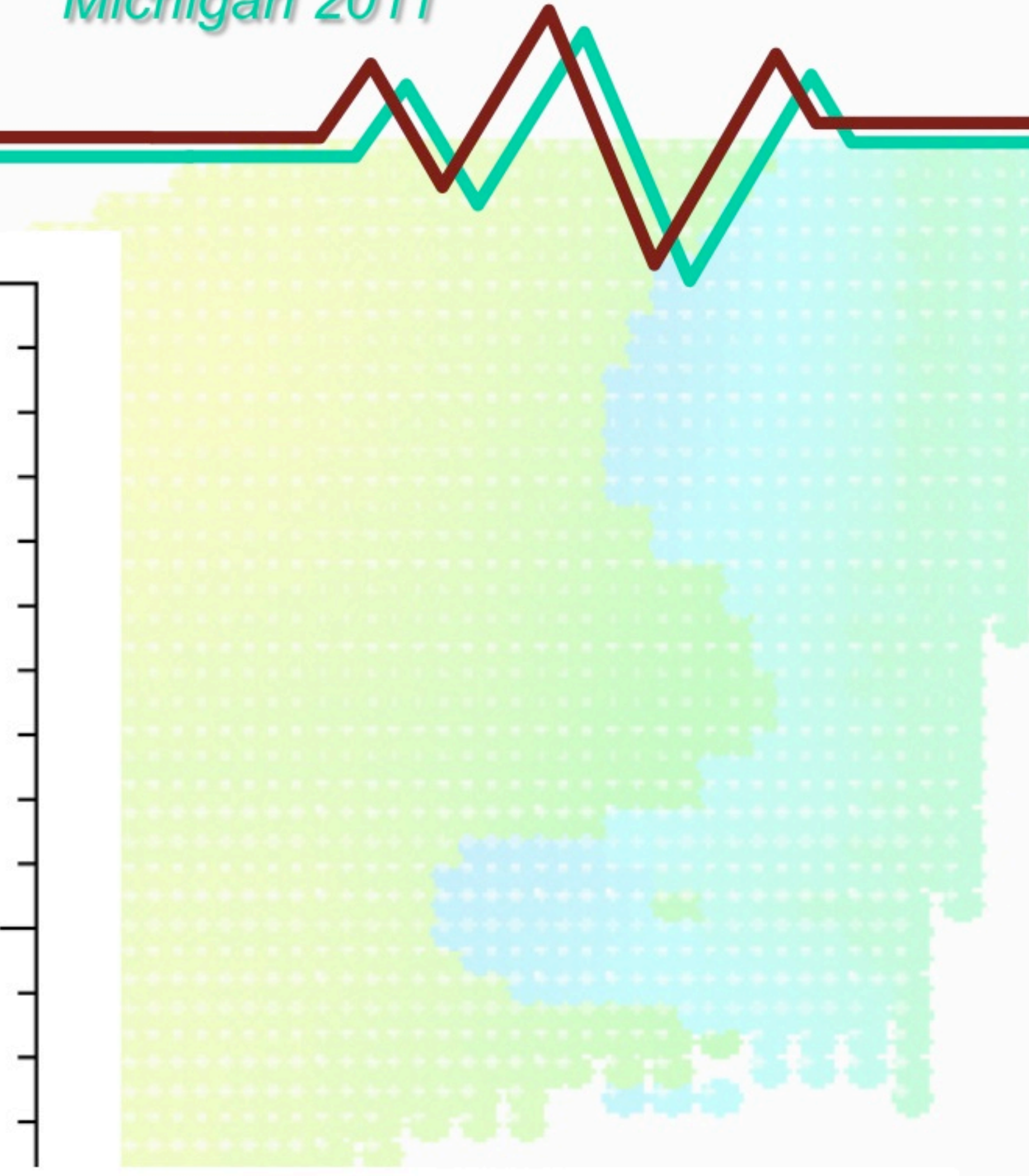


QRPA			FAM
$v_{crit}$	Size of $A, B$ matrices	Memory (in GB)	Memory (in GB)
$^{40}\text{Mg}$	$10^{-3}$	$32039 \times 32039$	16.4
	$10^{-4}$	$53386 \times 53386$	45.6
	$10^{-5}$	$53823 \times 53823$	46.35
	$10^{-10}$	$130936 \times 130936$	274.31
	$10^{-15}$	$189271 \times 189271$	473.18
	$10^{-20}$	$211159 \times 211159$	713.41
	0.572		
$^{100}\text{Zr}$	$10^{-3}$	$83970 \times 83970$	112.81
	$10^{-4}$	$140229 \times 140229$	314.63
	$10^{-5}$	$160633 \times 160633$	412.85
	$10^{-10}$	$189500 \times 189500$	574.56
	$10^{-15}$	$230274 \times 230274$	848.41
	$10^{-20}$	$230304 \times 230304$	848.64
	0.572		





# FAM QRPA RESULTS: $^{100}\text{Zr}$

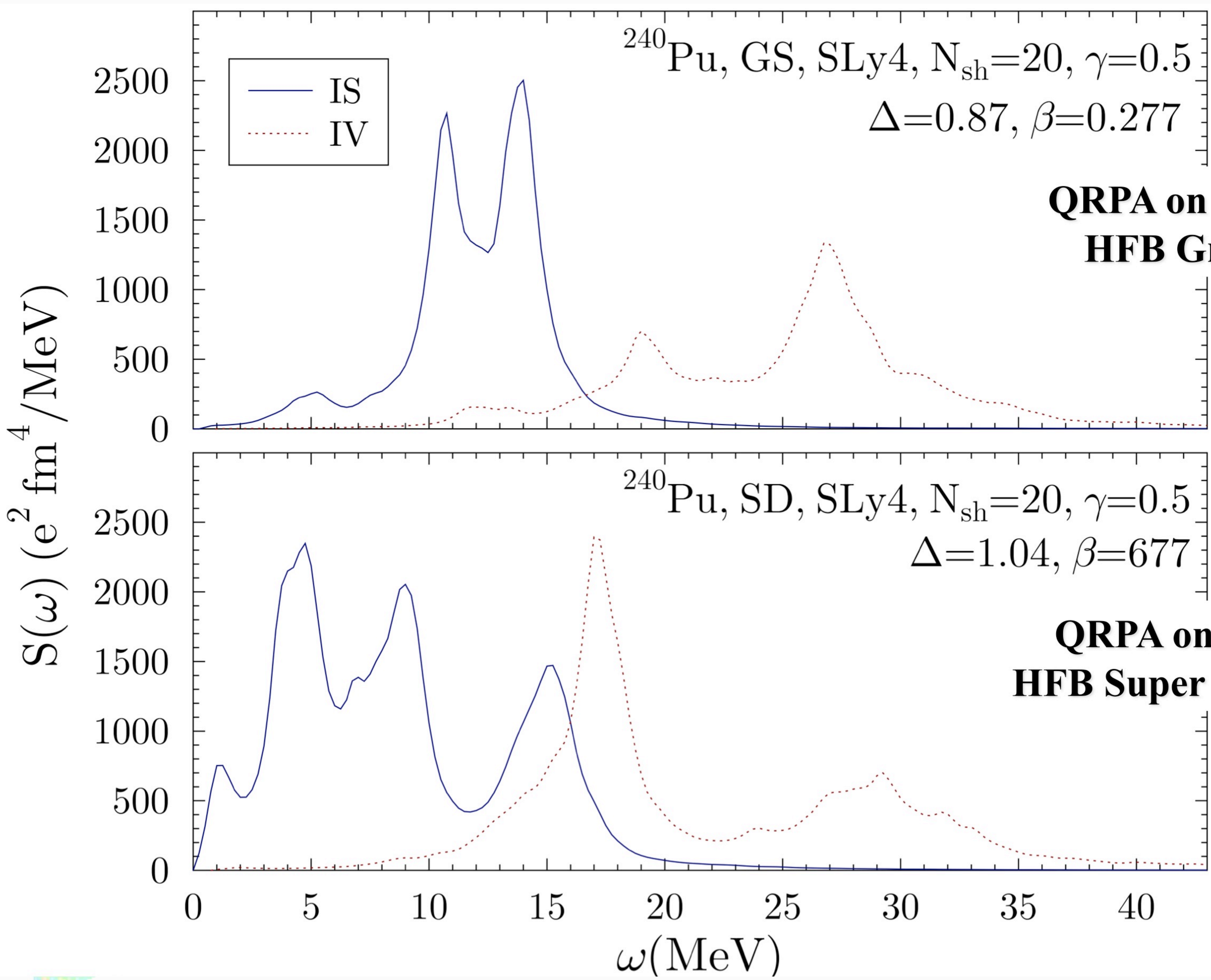


$$E_{\text{Skyrme}}[\rho] = \sum_{t=0,1} \int d^3r \{ C_t^\rho [\rho_{00}] \rho_{t0}^2(\mathbf{r}) + C_t^{\Delta\rho} \rho_{t0}(\mathbf{r}) \Delta\rho_{t0}(\mathbf{r}) + C_t^\tau [\rho_{t0}(\mathbf{r}) \tau_{t0}(\mathbf{r}) - \mathbf{j}_{t0}^2(\mathbf{r})] + C_t^s [\rho_{00}] s_{t0}^2(\mathbf{r}) + C_t^{\Delta s} s_{t0}(\mathbf{r}) \cdot \Delta s_{t0}(\mathbf{r}) + C_t^T [s_{00}(\mathbf{r}) \cdot \mathbf{T}_{t0}(\mathbf{r}) - \vec{J}_{t0}^2(\mathbf{r})] + C_t^{\nabla J} \times [\rho_{t0}(\mathbf{r}) \nabla \cdot \mathbf{J}_{t0}(\mathbf{r}) + s_{t0}(\mathbf{r}) \cdot \nabla \times \mathbf{j}_{t0}(\mathbf{r})] \}.$$

QRPA for the Zr-chain  
 K. Yoshida, Phys. Rev. C 82, 034324 (2010)



# FAM QRPA RESULTS: $^{240}\text{Pu}$



**QRPA on the top of the HFB Ground State**

**CLACULATIONS PERFORMED ON A LAPTOP**

**QRPA on the top of the HFB Super Deformed State**





# SUMMARY

## NEXT STEPS TO THE UNEDF FUNCTIONAL

- ❖ Optimization adding single-particle energies
- ❖ Optimization adding neutron drops ‘data’
- ❖ Optimization adding giant resonances data

Hopefully all Skyrme parameters will be constrained

- ❖ Optimization using DME functional

Considering exploring the separable pairing

Complete mass table QRPA calculations